Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/7.6.1-u-a+b-arccsch-c-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [178]. This is test number [202].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (178)	% 0. (0)
Mathematica	% 97.19 (173)	% 2.81 (5)
Maple	% 57.87 (103)	% 42.13 (75)
Maxima	% 28.65 (51)	% 71.35 (127)
Fricas	% 63.48 (113)	% 36.52 (65)
Sympy	% 7.87 (14)	% 92.13 (164)
Giac	% 25.84 (46)	% 74.16 (132)

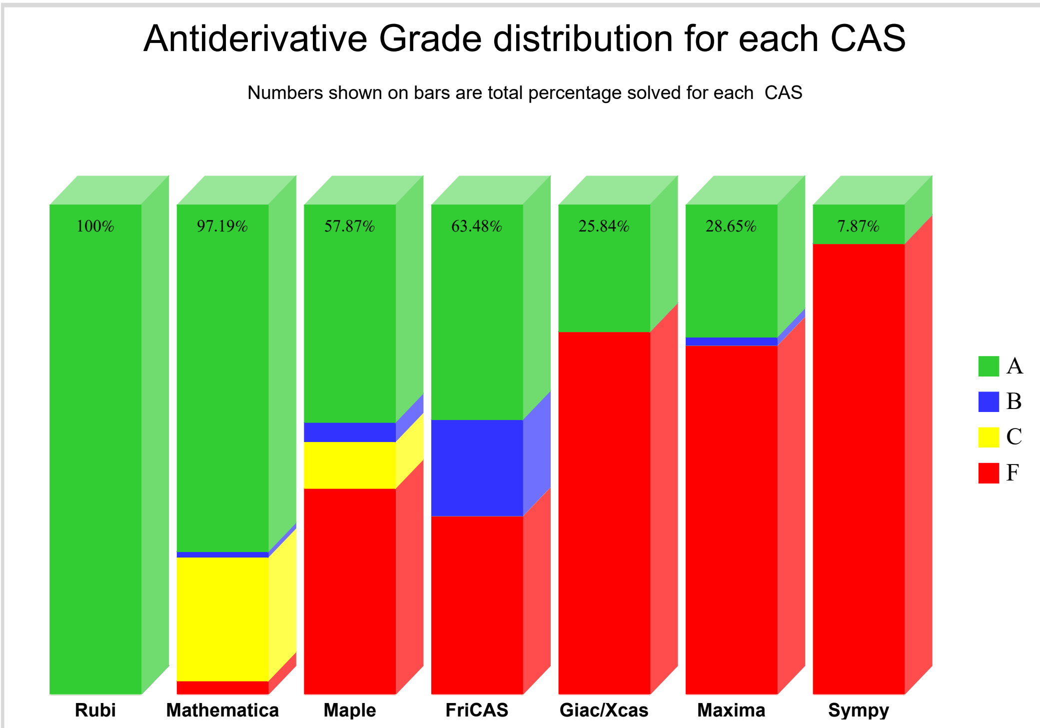
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

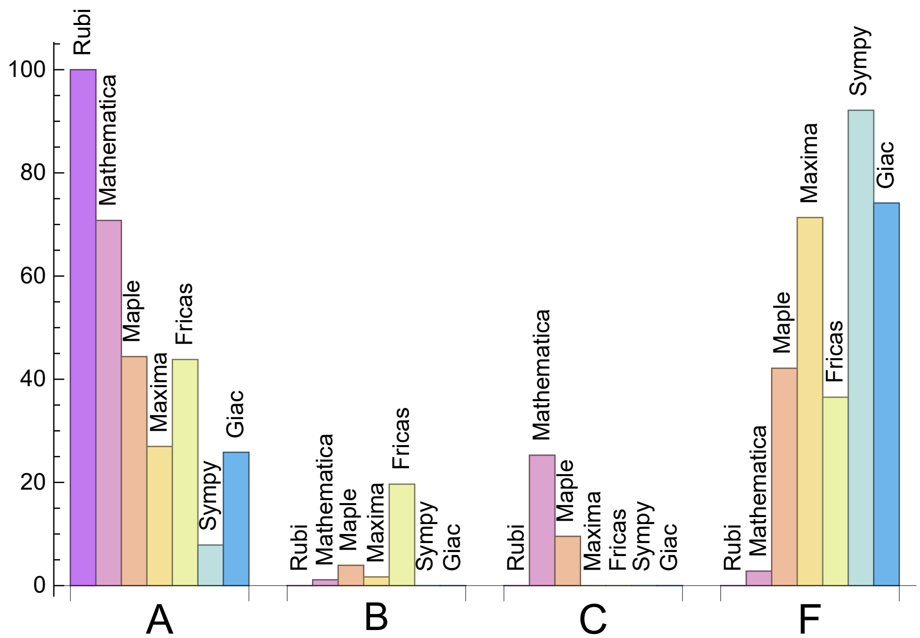
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	70.79	1.12	25.28	2.81
Maple	44.38	3.93	9.55	42.13
Maxima	26.97	1.69	0.	71.35
Fricas	43.82	19.66	0.	36.52
Sympy	7.87	0.	0.	92.13
Giac	25.84	0.	0.	74.16

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.52	217.7	0.74	132.5	1.
Mathematica	4.11	294.37	0.92	123.	0.88
Maple	0.66	389.77	1.23	75.	0.89
Maxima	0.75	145.41	1.25	123.	1.25
Fricas	2.31	658.65	3.87	266.	2.15
Sympy	0.2	2.57	0.09	0.	0.
Giac	0.	0.	0.	0.	0.

1.4 list of integrals that has no closed form antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {8, 99, 103, 104, 111}

Mathematica {8, 16, 18, 19, 24, 25, 26, 27, 28, 48, 51, 57, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 115, 116, 117, 118, 128}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

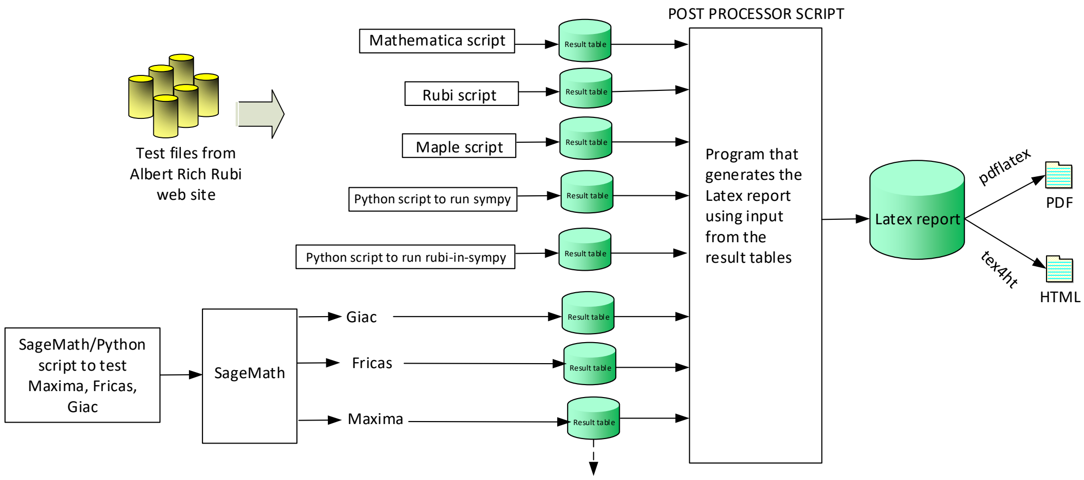
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 25, 27 }

C grade: { 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 126, 127, 128, 136, 137, 146, 155, 164 }

F grade: { 106, 114, 165, 166, 167}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 9, 10, 49, 50, 105, 112, 113 }

C grade: { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75 }

F grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 17, 20, 29, 33, 34, 35, 42, 43, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 168, 169, 170, 171, 172, 173, 177, 178

B grade: { 10, 12, 14 }

C grade: { }

F grade: { 8, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 39, 40, 41, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.5 FriCAS

A grade: { 2, 4, 10, 11, 12, 13, 14, 23, 32, 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 80, 81, 82, 83, 84, 85, 88, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 141, 142, 143, 144, 147, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178 }

B grade: { 1, 3, 5, 6, 7, 9, 15, 17, 20, 21, 22, 29, 30, 31, 44, 45, 46, 47, 49, 50, 78, 79, 89, 90, 91, 105, 112, 113, 140, 148, 149, 156, 157, 158, 176 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 126, 127, 136, 137, 145, 146, 154, 155, 163, 164, 165, 166, 167 }

2.1.6 Sympy

A grade: { 9, 33, 34, 35, 40, 42, 43, 61, 121, 124, 125, 141, 144, 177 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178 }

2.1.7 Giac

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	107	127	213	473	0	0
normalized size	1	1.	0.97	1.15	1.94	4.3	0.	0.
time (sec)	N/A	0.061	0.137	0.205	1.037	2.437	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	72	83	104	215	0	0
normalized size	1	1.	0.84	0.97	1.21	2.5	0.	0.
time (sec)	N/A	0.044	0.127	0.184	1.004	2.192	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	97	108	173	443	0	0
normalized size	1	1.	1.13	1.26	2.01	5.15	0.	0.
time (sec)	N/A	0.048	0.047	0.184	1.006	2.314	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	74	77	190	0	0
normalized size	1	1.	1.	1.19	1.24	3.06	0.	0.
time (sec)	N/A	0.027	0.094	0.194	1.01	2.266	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	85	87	130	414	0	0
normalized size	1	1.	1.37	1.4	2.1	6.68	0.	0.
time (sec)	N/A	0.035	0.05	0.171	0.986	2.325	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	65	47	154	0	0
normalized size	1	1.	1.32	1.71	1.24	4.05	0.	0.
time (sec)	N/A	0.013	0.023	0.197	1.007	2.208	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	36	66	320	0	0
normalized size	1	1.	1.47	1.2	2.2	10.67	0.	0.
time (sec)	N/A	0.021	0.06	0.198	1.016	2.257	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	47	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.04	0.198	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	40	62	43	135	36	0
normalized size	1	1.	1.33	2.07	1.43	4.5	1.2	0.
time (sec)	N/A	0.024	0.03	0.174	0.996	2.142	2.732	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	66	100	142	167	0	0
normalized size	1	1.	1.32	2.	2.84	3.34	0.	0.
time (sec)	N/A	0.037	0.033	0.182	0.998	2.203	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	59	75	76	171	0	0
normalized size	1	1.	1.02	1.29	1.31	2.95	0.	0.
time (sec)	N/A	0.043	0.046	0.197	0.996	2.137	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	120	198	196	0	0
normalized size	1	1.	1.05	1.62	2.68	2.65	0.	0.
time (sec)	N/A	0.048	0.042	0.195	1.015	2.126	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	69	83	99	197	0	0
normalized size	1	1.	0.87	1.05	1.25	2.49	0.	0.
time (sec)	N/A	0.051	0.061	0.193	0.978	2.12	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	88	139	250	225	0	0
normalized size	1	1.	0.9	1.42	2.55	2.3	0.	0.
time (sec)	N/A	0.064	0.072	0.197	1.052	2.177	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	603	0	0
normalized size	1	1.	1.16	0.	0.	5.74	0.	0.
time (sec)	N/A	0.119	0.205	0.187	0.	2.356	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	211	0	0	0	0	0
normalized size	1	1.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	1.366	0.184	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	87	0	111	524	0	0
normalized size	1	1.	1.61	0.	2.06	9.7	0.	0.
time (sec)	N/A	0.076	0.137	0.19	1.009	2.451	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	121	0	0	0	0	0
normalized size	1	1.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.222	0.207	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.131	0.201	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	0	105	296	0	0
normalized size	1	1.	1.43	0.	2.14	6.04	0.	0.
time (sec)	N/A	0.07	0.162	0.18	1.012	2.057	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	350	0	0
normalized size	1	1.	1.15	0.	0.	4.02	0.	0.
time (sec)	N/A	0.082	0.137	0.187	0.	2.114	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	106	0	0	385	0	0
normalized size	1	1.	1.06	0.	0.	3.85	0.	0.
time (sec)	N/A	0.108	0.184	0.186	0.	2.203	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	147	0	0	432	0	0
normalized size	1	1.	1.11	0.	0.	3.27	0.	0.
time (sec)	N/A	0.117	0.174	0.204	0.	2.229	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	271	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.975	0.178	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	548	0	0	0	0	0
normalized size	1	1.	2.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	7.484	0.199	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	171	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.457	0.191	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	246	0	0	0	0	0
normalized size	1	1.	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.324	0.191	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	198	0	0	0	0	0
normalized size	1	1.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.216	0.2	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	132	0	194	477	0	0
normalized size	1	1.	1.69	0.	2.49	6.12	0.	0.
time (sec)	N/A	0.099	0.232	0.186	0.992	2.312	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	182	0	0	576	0	0
normalized size	1	1.	1.48	0.	0.	4.68	0.	0.
time (sec)	N/A	0.106	0.311	0.204	0.	2.468	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	200	0	0	640	0	0
normalized size	1	1.	1.2	0.	0.	3.86	0.	0.
time (sec)	N/A	0.171	0.301	0.184	0.	2.566	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	277	0	0	747	0	0
normalized size	1	1.	1.36	0.	0.	3.66	0.	0.
time (sec)	N/A	0.181	0.336	0.183	0.	2.472	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	2.875	0.194	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	2.698	0.185	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.286	0.21	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.072	0.177	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.072	0.19	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	0.157	0.179	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	4.832	0.183	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	3.198	0.194	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.231	0.183	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.362	0.186	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.719	0.196	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	165	269	352	898	0	0
normalized size	1	1.	0.99	1.61	2.11	5.38	0.	0.
time (sec)	N/A	0.384	0.279	0.178	1.016	3.641	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	204	259	709	0	0
normalized size	1	1.	1.	1.67	2.12	5.81	0.	0.
time (sec)	N/A	0.259	0.181	0.218	1.029	3.171	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	99	115	117	479	0	0
normalized size	1	1.	1.22	1.42	1.44	5.91	0.	0.
time (sec)	N/A	0.161	0.195	0.22	1.002	2.65	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	36	66	320	0	0
normalized size	1	1.	1.47	1.2	2.2	10.67	0.	0.
time (sec)	N/A	0.022	0.048	0.169	0.988	2.385	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	506	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.39	0.729	0.566	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	134	208	0	741	0	0
normalized size	1	1.	1.37	2.12	0.	7.56	0.	0.
time (sec)	N/A	0.154	0.215	0.305	0.	2.683	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	204	963	0	1524	0	0
normalized size	1	1.	1.25	5.91	0.	9.35	0.	0.
time (sec)	N/A	0.292	0.462	0.273	0.	5.631	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	918	918	1094	2515	0	0	0	0
normalized size	1	1.	1.19	2.74	0.	0.	0.	0.
time (sec)	N/A	3.253	14.284	0.684	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	679	418	1964	0	0	0	0
normalized size	1	1.	0.62	2.89	0.	0.	0.	0.
time (sec)	N/A	2.519	1.605	0.32	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	926	840	0	0	0	0
normalized size	1	1.	2.16	1.96	0.	0.	0.	0.
time (sec)	N/A	0.708	13.992	0.301	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	18.568	6.829	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	7.771	4.661	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	380	1939	0	0	0	0
normalized size	1	1.	0.78	3.99	0.	0.	0.	0.
time (sec)	N/A	1.023	1.526	0.293	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	939	939	1098	2543	0	0	0	0
normalized size	1	1.	1.17	2.71	0.	0.	0.	0.
time (sec)	N/A	2.871	14.176	0.321	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	707	707	1012	1991	0	0	0	0
normalized size	1	1.	1.43	2.82	0.	0.	0.	0.
time (sec)	N/A	2.142	14.325	0.305	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	343	868	0	0	0	0
normalized size	1	1.	0.72	1.83	0.	0.	0.	0.
time (sec)	N/A	1.751	1.279	0.299	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	307	395	0	0	0	0
normalized size	1	1.	1.08	1.39	0.	0.	0.	0.
time (sec)	N/A	0.414	4.957	0.281	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	6.304	1.24	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	8.863	5.133	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	731	731	1042	2019	0	0	0	0
normalized size	1	1.	1.43	2.76	0.	0.	0.	0.
time (sec)	N/A	2.636	14.578	0.305	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	979	896	0	0	0	0
normalized size	1	1.	1.96	1.8	0.	0.	0.	0.
time (sec)	N/A	2.01	14.367	0.299	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	264	418	0	0	0	0
normalized size	1	1.	0.83	1.31	0.	0.	0.	0.
time (sec)	N/A	1.691	1.558	0.323	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	166	328	0	0	0	0
normalized size	1	1.	1.11	2.2	0.	0.	0.	0.
time (sec)	N/A	0.284	0.655	0.299	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	11.462	2.531	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	14.568	6.172	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	777	777	1108	2726	0	0	0	0
normalized size	1	1.	1.43	3.51	0.	0.	0.	0.
time (sec)	N/A	3.186	14.346	0.316	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	1076	2497	0	0	0	0
normalized size	1	1.	1.89	4.39	0.	0.	0.	0.
time (sec)	N/A	2.497	14.271	0.319	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	390	2107	0	0	0	0
normalized size	1	1.	0.99	5.36	0.	0.	0.	0.
time (sec)	N/A	2.191	2.42	0.308	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	892	2079	0	0	0	0
normalized size	1	1.	2.42	5.63	0.	0.	0.	0.
time (sec)	N/A	0.567	14.064	0.316	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	30.203	4.395	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	29.24	7.439	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	648	785	1217	3782	0	0	0	0
normalized size	1	1.21	1.88	5.84	0.	0.	0.	0.
time (sec)	N/A	1.035	14.295	0.326	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	138	211	390	680	0	0
normalized size	1	1.	0.64	0.99	1.82	3.18	0.	0.
time (sec)	N/A	0.132	0.233	0.196	1.022	3.32	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	119	171	306	617	0	0
normalized size	1	1.	0.71	1.02	1.83	3.69	0.	0.
time (sec)	N/A	0.109	0.167	0.18	1.01	3.17	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	135	126	200	544	0	0
normalized size	1	1.	1.17	1.1	1.74	4.73	0.	0.
time (sec)	N/A	0.056	0.263	0.209	1.02	2.959	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	89	107	113	493	0	0
normalized size	1	1.	0.98	1.18	1.24	5.42	0.	0.
time (sec)	N/A	0.068	0.13	0.194	0.975	2.762	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	122	123	234	0	0
normalized size	1	1.	0.62	1.12	1.13	2.15	0.	0.
time (sec)	N/A	0.079	0.085	0.2	0.994	2.359	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	93	140	178	294	0	0
normalized size	1	1.	0.59	0.89	1.13	1.86	0.	0.
time (sec)	N/A	0.103	0.122	0.197	1.001	2.604	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	109	158	223	350	0	0
normalized size	1	1.	0.53	0.77	1.09	1.71	0.	0.
time (sec)	N/A	0.122	0.146	0.198	1.018	2.703	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	114	152	238	378	0	0
normalized size	1	1.	0.56	0.75	1.17	1.85	0.	0.
time (sec)	N/A	0.161	0.248	0.2	1.006	3.051	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	97	134	185	323	0	0
normalized size	1	1.	0.61	0.84	1.16	2.03	0.	0.
time (sec)	N/A	0.134	0.208	0.197	1.023	3.343	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	77	115	128	266	0	0
normalized size	1	1.	0.53	0.79	0.88	1.82	0.	0.
time (sec)	N/A	0.106	0.095	0.184	1.008	2.777	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	93	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.291	0.134	0.222	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	138	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.558	0.231	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	182	286	535	891	0	0
normalized size	1	1.	0.7	1.1	2.06	3.43	0.	0.
time (sec)	N/A	0.258	0.353	0.19	1.009	4.209	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	149	217	387	792	0	0
normalized size	1	1.	0.76	1.1	1.96	4.02	0.	0.
time (sec)	N/A	0.128	0.229	0.184	1.037	3.813	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	134	189	258	747	0	0
normalized size	1	1.	0.79	1.11	1.52	4.39	0.	0.
time (sec)	N/A	0.139	0.214	0.187	1.035	3.269	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	123	191	205	743	0	0
normalized size	1	1.	0.75	1.16	1.25	4.53	0.	0.
time (sec)	N/A	0.143	0.263	0.191	1.029	3.013	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	126	191	236	382	0	0
normalized size	1	1.	0.67	1.01	1.25	2.02	0.	0.
time (sec)	N/A	0.16	0.217	0.191	1.01	2.454	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	152	223	313	481	0	0
normalized size	1	1.	0.61	0.9	1.26	1.93	0.	0.
time (sec)	N/A	0.197	0.259	0.207	1.04	2.55	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	159	214	329	506	0	0
normalized size	1	1.	0.64	0.86	1.32	2.02	0.	0.
time (sec)	N/A	0.235	0.403	0.19	1.055	2.787	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	123	182	247	410	0	0
normalized size	1	1.	0.61	0.9	1.22	2.02	0.	0.
time (sec)	N/A	0.156	0.302	0.173	1.026	2.725	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	148	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.421	0.44	0.383	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	187	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.424	0.902	0.368	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	1221	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	1.198	1.617	1.688	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	467	449	1103	0	0	0	0	0
normalized size	1	0.96	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	1.106	0.34	0.552	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	1055	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.849	0.493	1.143	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	425	425	387	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.862	0.957	0.48	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	1211	0	0	0	0	0
normalized size	1	1.	2.34	0.	0.	0.	0.	0.
time (sec)	N/A	1.06	1.604	1.497	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	591	571	1447	0	0	0	0	0
normalized size	1	0.97	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.282	6.017	0.585	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	553	535	1410	0	0	0	0	0
normalized size	1	0.97	2.55	0.	0.	0.	0.	0.
time (sec)	N/A	1.249	2.325	0.476	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	271	358	0	1288	0	0
normalized size	1	1.	1.95	2.58	0.	9.27	0.	0.
time (sec)	N/A	0.15	0.719	0.273	0.	2.732	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	515	515	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.153	43.207	0.586	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	756	756	1583	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	2.468	6.235	13.253	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	719	719	1442	0	0	0	0	0
normalized size	1	1.	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	1.242	2.58	1.7	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	713	713	1437	0	0	0	0	0
normalized size	1	1.	2.02	0.	0.	0.	0.	0.
time (sec)	N/A	2.187	3.487	1.997	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	758	758	1487	0	0	0	0	0
normalized size	1	1.	1.96	0.	0.	0.	0.	0.
time (sec)	N/A	2.246	2.876	19.474	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	694	676	2023	0	0	0	0	0
normalized size	1	0.97	2.91	0.	0.	0.	0.	0.
time (sec)	N/A	1.408	7.625	0.595	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	375	1914	0	2850	0	0
normalized size	1	1.	2.25	11.46	0.	17.07	0.	0.
time (sec)	N/A	0.215	1.435	0.281	0.	4.943	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	368	1884	0	2596	0	0
normalized size	1	1.	1.8	9.19	0.	12.66	0.	0.
time (sec)	N/A	0.205	0.883	0.279	0.	4.781	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	657	657	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.322	63.858	0.492	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2045	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	1.741	6.197	1.855	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2053	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	3.046	6.127	3.93	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1096	1096	2038	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	3.743	6.059	2.	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	413	345	0	0	4346	0	0
normalized size	1	1.	0.84	0.	0.	10.52	0.	0.
time (sec)	N/A	1.412	0.678	0.457	0.	20.553	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	337	0	0	3614	0	0
normalized size	1	1.	1.12	0.	0.	11.97	0.	0.
time (sec)	N/A	0.434	0.741	0.414	0.	9.544	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	278	0	0	2984	0	0
normalized size	1	1.	1.37	0.	0.	14.7	0.	0.
time (sec)	N/A	0.206	0.507	0.466	0.	5.439	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	5.393	0.452	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	5.632	0.453	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	9.078	0.448	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	2.584	0.464	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	1.701	0.449	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	237	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.427	0.597	0.539	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	314	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.638	0.666	0.451	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	318	0	0	4343	0	0
normalized size	1	1.	0.83	0.	0.	11.31	0.	0.
time (sec)	N/A	0.514	0.735	0.459	0.	19.924	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	314	0	0	3579	0	0
normalized size	1	1.	1.16	0.	0.	13.26	0.	0.
time (sec)	N/A	0.281	0.728	0.447	0.	10.5	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	6.294	0.449	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	5.661	0.447	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	9.293	0.448	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	3.482	0.459	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	5.537	0.491	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	15.47	0.458	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	291	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.575	0.682	0.453	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	643	643	372	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.866	0.824	0.453	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	339	0	0	3636	0	0
normalized size	1	1.	1.03	0.	0.	11.05	0.	0.
time (sec)	N/A	1.179	0.775	0.452	0.	11.059	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	280	0	0	3011	0	0
normalized size	1	1.	1.22	0.	0.	13.15	0.	0.
time (sec)	N/A	0.318	0.543	0.452	0.	5.885	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	223	0	0	2398	0	0
normalized size	1	1.	1.65	0.	0.	17.76	0.	0.
time (sec)	N/A	0.154	0.238	0.46	0.	3.932	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	1.675	0.454	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	22.798	0.455	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	6.254	0.451	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	1.039	0.482	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	0	0	0	0
normalized size	1	1.	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.2	0.455	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	239	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.486	0.626	0.491	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	311	0	0	3738	0	0
normalized size	1	1.	1.21	0.	0.	14.6	0.	0.
time (sec)	N/A	1.135	0.634	0.472	0.	6.069	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	233	0	0	2844	0	0
normalized size	1	1.	1.46	0.	0.	17.77	0.	0.
time (sec)	N/A	0.275	0.369	0.467	0.	4.387	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	122	0	0	822	0	0
normalized size	1	1.	1.49	0.	0.	10.02	0.	0.
time (sec)	N/A	0.111	0.159	0.463	0.	3.049	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	31.647	0.453	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	37.171	0.451	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	8.159	0.477	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	4.444	0.448	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.193	0.451	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	201	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	0.435	0.455	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	327	0	0	5065	0	0
normalized size	1	1.	1.3	0.	0.	20.18	0.	0.
time (sec)	N/A	1.229	0.556	0.466	0.	5.798	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	201	0	0	1627	0	0
normalized size	1	1.	1.19	0.	0.	9.63	0.	0.
time (sec)	N/A	0.268	0.284	0.5	0.	4.103	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	185	0	0	1439	0	0
normalized size	1	1.	1.28	0.	0.	9.99	0.	0.
time (sec)	N/A	0.144	0.236	0.496	0.	3.915	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	42.622	0.456	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	55.256	0.483	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	12.566	0.491	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	12.23	0.489	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	189	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.308	0.281	0.484	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	248	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	0.532	0.483	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	577	0	0	0	0	0	0
normalized size	1	0.97	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.575	0.212	0.362	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	379	360	0	0	0	0	0	0
normalized size	1	0.95	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.491	0.14	0.374	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	208	0	0	0	0	0	0
normalized size	1	0.95	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.22	0.105	0.188	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	1.791	0.519	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	2.717	0.501	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	1.084	0.484	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.107	0.483	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	1.448	0.486	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.717	0.48	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	214	0	0	846	0	0
normalized size	1	1.	0.54	0.	0.	2.14	0.	0.
time (sec)	N/A	2.344	0.309	0.483	0.	2.68	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	180	0	0	699	0	0
normalized size	1	1.	0.68	0.	0.	2.65	0.	0.
time (sec)	N/A	1.916	0.407	0.423	0.	2.777	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	133	141	0	0	571	0	0
normalized size	1	1.02	1.08	0.	0.	4.39	0.	0.
time (sec)	N/A	0.214	0.337	0.372	0.	2.858	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.386	0.464	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	3.309	0.454	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [69] had the largest ratio of [0.8571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.	12	0.417
2	A	4	3	1.	12	0.25
3	A	6	5	1.	12	0.417
4	A	3	3	1.	12	0.25
5	A	5	5	1.	12	0.417
6	A	2	2	1.	10	0.2
7	A	5	4	1.	8	0.5
8	A	6	6	1.	12	0.5
9	A	2	2	1.	12	0.167
10	A	4	4	1.	12	0.333
11	A	4	3	1.	12	0.25
12	A	5	4	1.	12	0.333
13	A	4	3	1.	12	0.25
14	A	6	4	1.	12	0.333
15	A	5	5	1.	14	0.357
16	A	8	6	1.	14	0.429
17	A	4	4	1.	12	0.333
18	A	7	5	1.	10	0.5
19	A	6	6	1.	14	0.429
20	A	4	3	1.	14	0.214
21	A	4	3	1.	14	0.214
22	A	5	5	1.	14	0.357
23	A	5	3	1.	14	0.214
24	A	10	10	1.	14	0.714
25	A	11	8	1.	14	0.571
26	A	7	7	1.	12	0.583
27	A	9	6	1.	10	0.6
28	A	7	7	1.	14	0.5
29	A	5	3	1.	14	0.214
30	A	6	6	1.	14	0.429
31	A	8	6	1.	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	10	6	1.	14	0.429
33	A	0	0	0.	0	0.
34	A	0	0	0.	0	0.
35	A	0	0	0.	0	0.
36	A	4	4	1.	14	0.286
37	A	6	6	1.	14	0.429
38	A	9	5	1.	14	0.357
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	3	3	1.	14	0.214
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	11	9	1.	16	0.562
45	A	10	9	1.	16	0.562
46	A	9	9	1.	14	0.643
47	A	5	4	1.	8	0.5
48	A	4	2	1.	16	0.125
49	A	7	7	1.	16	0.438
50	A	8	8	1.	16	0.5
51	A	31	16	1.	21	0.762
52	A	24	15	1.	19	0.79
53	A	15	11	1.	18	0.611
54	A	0	0	0.	0	0.
55	A	0	0	0.	0	0.
56	A	22	13	1.	18	0.722
57	A	27	17	1.	21	0.81
58	A	20	15	1.	21	0.714
59	A	14	13	1.	19	0.684
60	A	9	9	1.	18	0.5
61	A	0	0	0.	0	0.
62	A	0	0	0.	0	0.
63	A	23	15	1.	21	0.714

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	16	13	1.	21	0.619
65	A	11	11	1.	19	0.579
66	A	6	6	1.	18	0.333
67	A	0	0	0.	0	0.
68	A	0	0	0.	0	0.
69	A	31	18	1.	21	0.857
70	A	25	17	1.	21	0.81
71	A	19	14	1.	19	0.737
72	A	12	11	1.	18	0.611
73	A	0	0	0.	0	0.
74	A	0	0	0.	0	0.
75	A	19	14	1.21	18	0.778
76	A	7	7	1.	19	0.368
77	A	6	7	1.	19	0.368
78	A	5	5	1.	16	0.312
79	A	4	5	1.	19	0.263
80	A	4	5	1.	19	0.263
81	A	5	6	1.	19	0.316
82	A	6	6	1.	19	0.316
83	A	5	5	1.	19	0.263
84	A	5	5	1.	19	0.263
85	A	6	5	1.	17	0.294
86	A	11	11	1.	19	0.579
87	A	13	13	1.	19	0.684
88	A	7	8	1.	21	0.381
89	A	6	7	1.	18	0.389
90	A	6	7	1.	21	0.333
91	A	6	7	1.	21	0.333
92	A	5	6	1.	21	0.286
93	A	6	7	1.	21	0.333
94	A	5	6	1.	21	0.286
95	A	6	5	1.	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	12	13	1.	21	0.619
97	A	14	15	1.	21	0.714
98	A	25	12	1.	21	0.571
99	A	26	9	0.96	19	0.474
100	A	19	7	1.	18	0.389
101	A	19	7	1.	21	0.333
102	A	24	10	1.	21	0.476
103	A	31	14	0.97	21	0.667
104	A	29	12	0.97	21	0.571
105	A	7	6	1.	19	0.316
106	A	24	10	1.	21	0.476
107	A	51	15	1.	21	0.714
108	A	27	10	1.	21	0.476
109	A	47	11	1.	18	0.611
110	A	50	13	1.	21	0.619
111	A	33	13	0.97	21	0.619
112	A	6	7	1.	21	0.333
113	A	8	7	1.	19	0.368
114	A	28	11	1.	21	0.524
115	A	35	11	1.	21	0.524
116	A	63	12	1.	21	0.571
117	A	81	12	1.	18	0.667
118	A	12	12	1.	23	0.522
119	A	11	12	1.	23	0.522
120	A	9	9	1.	21	0.429
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	0	0	0.	0	0.
125	A	0	0	0.	0	0.
126	A	8	9	1.	23	0.391
127	A	9	10	1.	23	0.435

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	12	12	1.	23	0.522
129	A	10	10	1.	21	0.476
130	A	0	0	0.	0	0.
131	A	0	0	0.	0	0.
132	A	0	0	0.	0	0.
133	A	0	0	0.	0	0.
134	A	0	0	0.	0	0.
135	A	0	0	0.	0	0.
136	A	9	10	1.	23	0.435
137	A	10	10	1.	23	0.435
138	A	11	12	1.	23	0.522
139	A	10	12	1.	23	0.522
140	A	8	8	1.	21	0.381
141	A	0	0	0.	0	0.
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	A	8	9	1.	23	0.391
146	A	8	10	1.	23	0.435
147	A	10	11	1.	23	0.478
148	A	9	11	1.	23	0.478
149	A	4	4	1.	21	0.19
150	A	0	0	0.	0	0.
151	A	0	0	0.	0	0.
152	A	0	0	0.	0	0.
153	A	0	0	0.	0	0.
154	A	3	4	1.	20	0.2
155	A	7	9	1.	23	0.391
156	A	10	11	1.	23	0.478
157	A	7	8	1.	23	0.348
158	A	5	5	1.	21	0.238
159	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	0	0	0.	0	0.
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	7	8	1.	23	0.348
164	A	5	7	1.	20	0.35
165	A	6	7	0.97	23	0.304
166	A	6	7	0.95	23	0.304
167	A	5	6	0.95	21	0.286
168	A	0	0	0.	0	0.
169	A	0	0	0.	0	0.
170	A	0	0	0.	0	0.
171	A	0	0	0.	0	0.
172	A	0	0	0.	0	0.
173	A	0	0	0.	0	0.
174	A	16	11	1.	26	0.423
175	A	13	11	1.	26	0.423
176	A	8	9	1.02	26	0.346
177	A	0	0	0.	0	0.
178	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^6 \left(a + b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=110

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^6\sqrt{\frac{1}{c^2x^2}+1}}{42c}-\frac{5bx^4\sqrt{\frac{1}{c^2x^2}+1}}{168c^3}+\frac{5bx^2\sqrt{\frac{1}{c^2x^2}+1}}{112c^5}-\frac{5b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{112c^7}$$

```
[Out] (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(112*c^5) - (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsch[c*x
]))/7 - (5*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(112*c^7)
```

Rubi [A] time = 0.0608451, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6284, 266, 51, 63, 208}

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^6\sqrt{\frac{1}{c^2x^2}+1}}{42c}-\frac{5bx^4\sqrt{\frac{1}{c^2x^2}+1}}{168c^3}+\frac{5bx^2\sqrt{\frac{1}{c^2x^2}+1}}{112c^5}-\frac{5b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{112c^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(112*c^5) - (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsch[c*x
]))/7 - (5*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(112*c^7)
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)dx=\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^5}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{7c}$$

$$=\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{x^4\sqrt{1+\frac{x}{c^2}}}dx,x,\frac{1}{x^2}\right)}{14c}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(5b)\,\mathrm{Subst}\left(\int\frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}dx,x,\frac{1}{x^2}\right)}{84c^3}$$

$$=-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(5b)\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}dx,x,\right.}{112c^5}$$

$$=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(5b)\,\mathrm{Subst}}{}$$

$$=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(5b)\,\mathrm{Subst}}{}$$

$$=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{5b\tanh^{-1}\left(\right.}{112}$$

Mathematica [A] time = 0.136911, size = 107, normalized size = 0.97

$$\frac{ax^7}{7}+b\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(-\frac{5x^4}{168c^3}+\frac{5x^2}{112c^5}+\frac{x^6}{42c}\right)-\frac{5b\log\left(x\left(\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1\right)\right)}{112c^7}+\frac{1}{7}bx^7\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) - (5*x^4)/(1
68*c^3) + x^6/(42*c)) + (b*x^7*ArcCsch[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(1 +
c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

Maple [A] time = 0.205, size = 127, normalized size = 1.2

$$\frac{1}{c^7}\left(\frac{c^7x^7a}{7}+b\left(\frac{c^7x^7\mathrm{arccsch}\,(cx)}{7}+\frac{1}{336\,cx}\sqrt{c^2x^2+1}\left(8\,c^5x^5\sqrt{c^2x^2+1}-10\,c^3x^3\sqrt{c^2x^2+1}+15\,cx\sqrt{c^2x^2+1}-15\,\mathrm{Arcsinh}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^7*(1/7*c^7*x^7*a+b*(1/7*c^7*x^7*arccsch(c*x)+1/336*(c^2*x^2+1)^(1/2)*(8
*c^5*x^5*(c^2*x^2+1)^(1/2)-10*c^3*x^3*(c^2*x^2+1)^(1/2)+15*c*x*(c^2*x^2+1)^
(1/2)-15*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.03671, size = 213, normalized size = 1.94

$$\frac{1}{7}\,ax^7+\frac{1}{672}\left(96\,x^7\,\mathrm{arcsch}\,(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3\,c^6\left(\frac{1}{c^2x^2}+1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7 + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 4
0*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1
)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(s
qrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b
```

Fricas [B] time = 2.4366, size = 473, normalized size = 4.3

$$\frac{48\,ac^7x^7+48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+15\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)+48\left(bc^7x^7-b\right.}{336\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 + 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
+ 1) - 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 15*b*log
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 48*(b*c^7*x^7 - b*c^7)*log((c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*b*c^6*x^6 - 10*b*c^4*x^4 +
15*b*c^2*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\left(a+b\operatorname{acsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**6*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcsch}(cx)+a)x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^6, x)
```

3.2 $\int x^5 \left(a + b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=86

$$\frac{1}{6}x^6\left(a+b\,\text{csch}^{-1}(cx)\right)+\frac{bx^5\sqrt{\frac{1}{c^2x^2}+1}}{30c}-\frac{2bx^3\sqrt{\frac{1}{c^2x^2}+1}}{45c^3}+\frac{4bx\sqrt{\frac{1}{c^2x^2}+1}}{45c^5}$$

```
[Out] (4*b*Sqrt[1 + 1/(c^2*x^2)]*x)/(45*c^5) - (2*b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsch[c*x]))/
6
```

Rubi [A] time = 0.0441296, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6284, 271, 191}

$$\frac{1}{6}x^6\left(a+b\,\text{csch}^{-1}(cx)\right)+\frac{bx^5\sqrt{\frac{1}{c^2x^2}+1}}{30c}-\frac{2bx^3\sqrt{\frac{1}{c^2x^2}+1}}{45c^3}+\frac{4bx\sqrt{\frac{1}{c^2x^2}+1}}{45c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*Sqrt[1 + 1/(c^2*x^2)]*x)/(45*c^5) - (2*b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsch[c*x]))/
6
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)dx &= \frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{6c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c}+\frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(2b)\int\frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{15c^3}\\
&= -\frac{2b\sqrt{1+\frac{1}{c^2x^2}}x^3}{45c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c}+\frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(4b)\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{45c^5}\\
&= \frac{4b\sqrt{1+\frac{1}{c^2x^2}}x}{45c^5}-\frac{2b\sqrt{1+\frac{1}{c^2x^2}}x^3}{45c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c}+\frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.127229, size = 72, normalized size = 0.84

$$\frac{ax^6}{6}+b\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(-\frac{2x^3}{45c^3}+\frac{4x}{45c^5}+\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) - (2*x^3)/(45*c
^3) + x^5/(30*c)) + (b*x^6*ArcCsch[c*x])/6
```

Maple [A] time = 0.184, size = 83, normalized size = 1.

$$\frac{1}{c^6}\left(\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\operatorname{arccsch}(cx)}{6}+\frac{\left(c^2x^2+1\right)\left(3c^4x^4-4c^2x^2+8\right)}{90cx}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arccsch(c*x)+1/90*(c^2*x^2+1)*(3*c^4*x^
4-4*c^2*x^2+8)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.00425, size = 104, normalized size = 1.21

$$\frac{1}{6}\,ax^{6}+\frac{1}{90}\left(15\,x^{6}\,\mathrm{arcsch}\left(cx\right)+\frac{3\,c^{4}x^{5}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{5}{2}}-10\,c^{2}x^{3}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c^{5}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2)
- 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b
```

Fricas [A] time = 2.19157, size = 215, normalized size = 2.5

$$\frac{15\,bc^{5}x^{6}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+15\,ac^{5}x^{6}+\left(3\,bc^{4}x^{5}-4\,bc^{2}x^{3}+8\,bx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}{90\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^5*x^6*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 15*
a*c^5*x^6 + (3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2
)))/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{5}\left(a+b\,\mathrm{acsch}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5, x)
```

3.3 $\int x^4 \left(a + b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=86

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bx^4\sqrt{\frac{1}{c^2x^2}+1}}{20c}-\frac{3bx^2\sqrt{\frac{1}{c^2x^2}+1}}{40c^3}+\frac{3b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{40c^5}$$

```
[Out] (-3*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^4)/(
20*c) + (x^5*(a + b*ArcCsch[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])
/(40*c^5)
```

Rubi [A] time = 0.047877, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6284, 266, 51, 63, 208}

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bx^4\sqrt{\frac{1}{c^2x^2}+1}}{20c}-\frac{3bx^2\sqrt{\frac{1}{c^2x^2}+1}}{40c^3}+\frac{3b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{40c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcCsch[c*x]),x]
```


```
[Out] (-3*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^4)/(
20*c) + (x^5*(a + b*ArcCsch[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])
/(40*c^5)
```


Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)dx &= \frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{5c}\\
&= \frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(3b)\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{40c^3}\\
&= -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(3b)\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\right.}{80c^5}\\
&= -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(3b)\,\mathrm{Subst}\left(\int\frac{1}{-c^2+c^2x^2}\,dx,x,\right.}{40c^3}\\
&= -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{3b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{40c^5}
\end{aligned}
$$

Mathematica [A] time = 0.0468538, size = 97, normalized size = 1.13

$$\frac{ax^5}{5}+b\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(\frac{x^4}{20c}-\frac{3x^2}{40c^3}\right)+\frac{3b\log\left(x\left(\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1\right)\right)}{40c^5}+\frac{1}{5}bx^5\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) + x^4/(20*c)
) + (b*x^5*ArcCsch[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])
])/(40*c^5)
```

Maple [A] time = 0.184, size = 108, normalized size = 1.3

$$\frac{1}{c^5}\left(\frac{c^5x^5a}{5}+b\left(\frac{c^5x^5\operatorname{arccsch}(cx)}{5}+\frac{1}{40\,cx}\sqrt{c^2x^2+1}\left(2\,c^3x^3\sqrt{c^2x^2+1}-3\,cx\sqrt{c^2x^2+1}+3\operatorname{Arcsinh}(cx)\right)\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a+b*(1/5*c^5*x^5*arccsch(c*x)+1/40*(c^2*x^2+1)^(1/2)*(2*
c^3*x^3*(c^2*x^2+1)^(1/2)-3*c*x*(c^2*x^2+1)^(1/2)+3*arcsinh(c*x))/((c^2*x^2
+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.00616, size = 173, normalized size = 2.01

$$\frac{1}{5}\,ax^5+\frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) +
c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) -
 1)/c^4)/c)*b
```

Fricas [B] time = 2.3142, size = 443, normalized size = 5.15

$$\frac{8\,ac^5x^5 + 8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - 3\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + 8\left(bc^5x^5 - bc^5\right)\mathrm{l}}{40\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 + 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*b*log(c*x*
sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 8*(b*c^5*x^5 - b*c^5)*log((c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^4*x^4 - 3*b*c^2*x^2)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(a + b\,\mathrm{acsch}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcsch}\,(cx) + a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4, x)
```

3.4 $\int x^3 \left(a + b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=62

$$\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{6c^3}$$

```
[Out] -(b*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c)
 + (x^4*(a + b*ArcCsch[c*x]))/4
```

Rubi [A] time = 0.0274502, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6284, 271, 191}

$$\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -(b*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c)
 + (x^4*(a + b*ArcCsch[c*x]))/4
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)dx &= \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{4c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{6c^3}\\
&= -\frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.094009, size = 62, normalized size = 1.

$$\frac{ax^4}{4}+b\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(\frac{x^3}{12c}-\frac{x}{6c^3}\right)+\frac{1}{4}bx^4\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^4)/4 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*(-x/(6*c^3) + x^3/(12*c)) + (b*
x^4*ArcCsch[c*x])/4
```

Maple [A] time = 0.194, size = 74, normalized size = 1.2

$$\frac{1}{c^4}\left(\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\operatorname{arccsch}(cx)}{4}+\frac{\left(c^2x^2+1\right)\left(c^2x^2-2\right)}{12\,cx}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arccsch(c*x)+1/12*(c^2*x^2+1)*(c^2*x^2-
2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.01002, size = 77, normalized size = 1.24

$$\frac{1}{4}ax^4 + \frac{1}{12}\left(3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b
```

Fricas [A] time = 2.26618, size = 190, normalized size = 3.06

$$\frac{3\,bc^3x^4 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 3\,ac^3x^4 + \left(bc^2x^3 - 2\,bx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 3*a*
c^3*x^4 + (b*c^2*x^3 - 2*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + b\operatorname{acsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcsch}\,(cx) + a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3, x)
```

3.5 $\int x^2 \left(a + b\,\mathrm{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=62

$$\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}-\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}$$

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsch[c*x]))/3 - (b*Arc
Tanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rubi [A] time = 0.0350997, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6284, 266, 51, 63, 208}

$$\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}-\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsch[c*x]))/3 - (b*Arc
Tanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)dx &= \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{3c}\\
&= \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\,\mathrm{Subst}\left(\int\frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}
\end{aligned}$$

Mathematica [A] time = 0.049858, size = 85, normalized size = 1.37

$$\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{6c}-\frac{b\log\left(x\left(\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1\right)\right)}{6c^3}+\frac{1}{3}bx^3\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^3)/3 + (b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsch[c*
x])/3 - (b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A] time = 0.171, size = 87, normalized size = 1.4

$$\frac{1}{c^3}\left(\frac{ac^3x^3}{3}+b\left(\frac{c^3x^3\operatorname{arccsch}(cx)}{3}-\frac{1}{6\,cx}\sqrt{c^2x^2+1}\left(-cx\sqrt{c^2x^2+1}+\operatorname{Arcsinh}(cx)\right)\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^3*(1/3*a*c^3*x^3+b*(1/3*c^3*x^3*arccsch(c*x)-1/6*(c^2*x^2+1)^(1/2)*(-c*
x*(c^2*x^2+1)^(1/2)+arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 0.985773, size = 130, normalized size = 2.1

$$\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^
2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x
^2) + 1) - 1)/c^2)/c)*b
```

Fricas [B] time = 2.32475, size = 414, normalized size = 6.68

$$\frac{2\,ac^3x^3 + bc^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2\,bc^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 2\,bc^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) + b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + 2\,(}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 + b*c^2*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*b*c^3*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*b*c^3*log(c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) - c*x - 1) + b*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x)
 + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
)/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,(a + b\,\mathrm{acsch}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcsch}\,(cx) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2, x)
```

3.6 $\int x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=38

$$\frac{1}{2}x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2c}$$

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsch[c*x]))/2
```

Rubi [A] time = 0.0128323, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6284, 191}

$$\frac{1}{2}x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsch[c*x]))/2
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)dx=\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{2c}$$
$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

Mathematica [A] time = 0.0229745, size = 50, normalized size = 1.32

$$\frac{ax^2}{2}+\frac{bx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^2)/2 + (b*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsch[c*x]
)/2
```

Maple [A] time = 0.197, size = 65, normalized size = 1.7

$$\frac{1}{c^2}\left(\frac{c^2x^2a}{2}+b\left(\frac{c^2x^2\operatorname{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\,cx}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a+b*(1/2*c^2*x^2*arccsch(c*x)+1/2/((c^2*x^2+1)/c^2/x^2)^
(1/2)/c/x*(c^2*x^2+1)))
```

Maxima [A] time = 1.00651, size = 47, normalized size = 1.24

$$\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b
```

Fricas [B] time = 2.20757, size = 154, normalized size = 4.05

$$\frac{bcx^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+acx^2+bx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2 +
 b*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{acsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcsch}(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x, x)
```

3.7 $\int \left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax+\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+bx\text{csch}^{-1}(cx)$$

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rubi [A] time = 0.0209549, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6278, 266, 63, 208}

$$ax+\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+bx\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 6278

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\operatorname{csch}^{-1}(cx)\right) dx &= ax+b\int \operatorname{csch}^{-1}(cx)\,dx \\
&= ax+bx\operatorname{csch}^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x}\,dx}{c} \\
&= ax+bx\operatorname{csch}^{-1}(cx)-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c} \\
&= ax+bx\operatorname{csch}^{-1}(cx)-(bc)\,\mathrm{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right) \\
&= ax+bx\operatorname{csch}^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.0595727, size = 44, normalized size = 1.47

$$
ax+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+bx\operatorname{csch}^{-1}(cx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*Sqrt[1 + 1/(c^2*x^2)]*x*ArcSinh[c*x])/Sqrt[1 +
c^2*x^2]
```

Maple [A] time = 0.198, size = 36, normalized size = 1.2

$$
ax+bx\operatorname{arccsch}(cx)+\frac{b}{c}\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsch(c*x),x)
```

```
[Out] a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

Maxima [A] time = 1.01569, size = 66, normalized size = 2.2

$$ax + \frac{\left(2\,cx\operatorname{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b/c
```

Fricas [B] time = 2.25656, size = 320, normalized size = 10.67

$$\frac{acx + bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (bcx - bc)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{acsch}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsch(c*x),x)
```

```
[Out] Integral(a + b*acsch(c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b\,\mathrm{arcsch}\,(cx) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arccsch(c*x) + a, x)
```

3.8 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=56

$$\frac{1}{2}b\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b}-\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] -(a + b*ArcCsch[c*x])^2/(2*b) - (a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[
c*x])] + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/2
```

Rubi [A] time = 0.0898566, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6282, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}b\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b}-\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^2/(2*b) - (a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*
x])] - (b*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rule 6282

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2b} + 2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\,\text{csch}^{-1}(cx)}\right) + b\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{c}\right.$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\,\text{csch}^{-1}(cx)}\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2}\right.$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\,\text{csch}^{-1}(cx)}\right) - \frac{1}{2}b\,\text{Li}_2\left(e^{2\,\text{csch}^{-1}(cx)}\right)$$

Mathematica [A] time = 0.0404819, size = 47, normalized size = 0.84

$$\frac{1}{2}b\left(\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)-\text{csch}^{-1}(cx)\left(\text{csch}^{-1}(cx)+2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right)\right)+a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])
])) + PolyLog[2, E^(-2*ArcCsch[c*x])]))/2
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{a+b\text{arccsch}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x,x)
```

```
[Out] int((a+b*arccsch(c*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(4c^2\int\frac{x^2\log(x)}{c^2x^3+x}\,dx-2c^2\int\frac{x\log(x)}{c^2x^2+\left(c^2x^2+1\right)^{\frac{3}{2}}+1}\,dx-\left(\log\left(c^2x^2+1\right)-2\log(x)\right)\log(c)+\log\left(c^2x^2+1\right)\log(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log(
x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x))*
log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) + 2*
integrate(log(x)/(c^2*x^3 + x), x))*b + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsch}(cx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x, x)
```

3.9 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=30

$$bc\sqrt{\frac{1}{c^2x^2}+1}-\frac{a+b\text{csch}^{-1}(cx)}{x}$$

```
[Out] b*c*Sqrt[1 + 1/(c^2*x^2)] - (a + b*ArcCsch[c*x])/x
```

Rubi [A] time = 0.0238494, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6284, 261}

$$bc\sqrt{\frac{1}{c^2x^2}+1}-\frac{a+b\text{csch}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^2,x]
```

```
[Out] b*c*Sqrt[1 + 1/(c^2*x^2)] - (a + b*ArcCsch[c*x])/x
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{x} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^3}\,dx}{c}$$
$$= bc\sqrt{1+\frac{1}{c^2x^2}} - \frac{a+b\text{csch}^{-1}(cx)}{x}$$

Mathematica [A] time = 0.0297578, size = 40, normalized size = 1.33

$$-\frac{a}{x} + bc\sqrt{\frac{c^2x^2+1}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^2,x]
```

```
[Out] -(a/x) + b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/x
```

Maple [B] time = 0.174, size = 62, normalized size = 2.1

$$c\left(-\frac{a}{cx} + b\left(-\frac{\text{arccsch}\,(cx)}{cx} + \frac{c^2x^2+1}{c^2x^2}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2,x)
```

```
[Out] c*(-a/c/x+b*(-1/c/x*arccsch(c*x)+1/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2
*x^2+1)))
```

Maxima [A] time = 0.995552, size = 43, normalized size = 1.43

$$\left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\text{arcsch}\,(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b - a/x
```

Fricas [B] time = 2.14161, size = 135, normalized size = 4.5

$$\frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) - a)/x
```

Sympy [A] time = 2.73184, size = 36, normalized size = 1.2

$$\begin{cases} -\frac{a}{x} + bc\sqrt{1 + \frac{1}{c^2x^2}} - \frac{b\,\operatorname{acsch}(cx)}{x} & \text{for}\ c \neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*sqrt(1 + 1/(c**2*x**2)) - b*acsch(c*x)/x, Ne(c, 0)),
(-(a + zoo*b)/x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\operatorname{arcsch}(cx) + a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^2, x)
```

3.10 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=50

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{2x^2}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x}-\frac{1}{4}bc^2\mathrm{csch}^{-1}(cx)$$

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*ArcCsch[c*x])/4 - (a + b*ArcCsch
[c*x])/(2*x^2)
```

Rubi [A] time = 0.0372564, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6284, 335, 321, 215}

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{2x^2}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x}-\frac{1}{4}bc^2\mathrm{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^3,x]
```


```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*ArcCsch[c*x])/4 - (a + b*ArcCsch
[c*x])/(2*x^2)
```


Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2}-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}\,dx}{2c}\\ &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2}+\frac{b\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}\\ &= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x}-\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2}-\frac{1}{4}(bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\ &= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2\operatorname{csch}^{-1}(cx)-\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0331019, size = 66, normalized size = 1.32

$$-\frac{a}{2x^2}+\frac{bc\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2\sinh^{-1}\left(\frac{1}{cx}\right)-\frac{b\operatorname{csch}^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsch[c*x])/(
2*x^2) - (b*c^2*ArcSinh[1/(c*x)])/4
```

Maple [B] time = 0.182, size = 100, normalized size = 2.

$$c^{2}\left(-\frac{a}{2c^{2}x^{2}}+b\left(-\frac{\operatorname{arccsch}(cx)}{2c^{2}x^{2}}-\frac{1}{4c^{3}x^{3}}\sqrt{c^{2}x^{2}+1}\left(\operatorname{Artanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)c^{2}x^{2}-\sqrt{c^{2}x^{2}+1}\right)\frac{1}{\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3,x)
```

```
[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arccsch(c*x)-1/4*(c^2*x^2+1)^(1/2)*(arc
tanh(1/(c^2*x^2+1)^(1/2))*c^2*x^2-(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^
(1/2)/c^3/x^3))
```

Maxima [B] time = 0.997988, size = 142, normalized size = 2.84

$$\frac{1}{8}b\left(\frac{\frac{2c^{4}x\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c^{2}x^{2}\left(\frac{1}{c^{2}x^{2}}+1\right)-1}-c^{3}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}+1}+1\right)+c^{3}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}+1}-1\right)}{c}-\frac{4\operatorname{arcsch}(cx)}{x^{2}}\right)-\frac{a}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3
*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1
))/c - 4*arccsch(c*x)/x^2) - 1/2*a/x^2
```

Fricas [A] time = 2.2034, size = 167, normalized size = 3.34

$$\frac{bcx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-\left(bc^{2}x^{2}+2b\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)-2a}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b*c^2*x^2 + 2*b)*log((c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^3, x)
```

3.11 $\int \frac{a+b\textbf{csch}^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=58

$$-\frac{a+b\text{csch}^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(\frac{1}{c^2x^2}+1\right)^{3/2}-\frac{1}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}$$

```
[Out] -(b*c^3*Sqrt[1 + 1/(c^2*x^2)])/3 + (b*c^3*(1 + 1/(c^2*x^2))^(3/2))/9 - (a +
 b*ArcCsch[c*x])/(3*x^3)
```

Rubi [A] time = 0.0428057, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6284, 266, 43}

$$-\frac{a+b\text{csch}^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(\frac{1}{c^2x^2}+1\right)^{3/2}-\frac{1}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^4,x]
```

```
[Out] -(b*c^3*Sqrt[1 + 1/(c^2*x^2)])/3 + (b*c^3*(1 + 1/(c^2*x^2))^(3/2))/9 - (a +
 b*ArcCsch[c*x])/(3*x^3)
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}\,dx}{3c}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}+\frac{b\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}+\frac{b\,\mathrm{Subst}\left(\int \left(-\frac{c^2}{\sqrt{1+\frac{x}{c^2}}}+c^2\sqrt{1+\frac{x}{c^2}}\right)dx,x,\frac{1}{x^2}\right)}{6c}\\
&= -\frac{1}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1+\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0455896, size = 59, normalized size = 1.02

$$-\frac{a}{3x^3}+b\left(\frac{c}{9x^2}-\frac{2c^3}{9}\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-\frac{b\operatorname{csch}^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^4,x]
```

```
[Out] -a/(3*x^3) + b*((-2*c^3)/9 + c/(9*x^2))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*
ArcCsch[c*x])/(3*x^3)
```

Maple [A] time = 0.197, size = 75, normalized size = 1.3

$$c^3\left(-\frac{a}{3\,c^3x^3}+b\left(-\frac{\operatorname{arccsch}(cx)}{3\,c^3x^3}-\frac{\left(c^2x^2+1\right)\left(2\,c^2x^2-1\right)}{9\,c^4x^4}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^4,x)
```

```
[Out] c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*arccsch(c*x)-1/9*(c^2*x^2+1)*(2*c^2*x^2
-1)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4))
```

Maxima [A] time = 0.995903, size = 76, normalized size = 1.31

$$\frac{1}{9}\,b\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arcsch}(cx)}{x^3}\right)-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*ar
ccsch(c*x)/x^3) - 1/3*a/x^3
```

Fricas [A] time = 2.13714, size = 171, normalized size = 2.95

$$-\frac{3\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(2\,bc^3x^3-bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+3\,a}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^3*x^3
 - b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 3*a)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{acsch}(cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^4, x)
```

3.12 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=74

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4x^4}-\frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}}{32x}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{16x^3}+\frac{3}{32}bc^4\mathrm{csch}^{-1}(cx)$$

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(32*
x) + (3*b*c^4*ArcCsch[c*x])/32 - (a + b*ArcCsch[c*x])/(4*x^4)
```

Rubi [A] time = 0.0480371, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6284, 335, 321, 215}

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4x^4}-\frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}}{32x}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{16x^3}+\frac{3}{32}bc^4\mathrm{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^5,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(32*
x) + (3*b*c^4*ArcCsch[c*x])/32 - (a + b*ArcCsch[c*x])/(4*x^4)
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^6}\,dx}{4c}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}+\frac{b\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4c}\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}-\frac{1}{16}(3bc)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}+\frac{1}{32}\left(3bc^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x}+\frac{3}{32}bc^4\operatorname{csch}^{-1}(cx)-\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}
\end{aligned}
$$

Mathematica [A] time = 0.041736, size = 78, normalized size = 1.05

$$
-\frac{a}{4x^4}+b\left(\frac{c}{16x^3}-\frac{3c^3}{32x}\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\frac{3}{32}bc^4\sinh^{-1}\left(\frac{1}{cx}\right)-\frac{b\operatorname{csch}^{-1}(cx)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^5,x]
```

```
[Out] -a/(4*x^4) + b*(c/(16*x^3) - (3*c^3)/(32*x))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]
- (b*ArcCsch[c*x])/(4*x^4) + (3*b*c^4*ArcSinh[1/(c*x)])/32
```

Maple [A] time = 0.195, size = 120, normalized size = 1.6

$$c^4\left(-\frac{a}{4\,c^4x^4}+b\left(-\frac{\operatorname{arccsch}(cx)}{4\,c^4x^4}+\frac{1}{32\,c^5x^5}\sqrt{c^2x^2+1}\left(3\operatorname{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4-3\,c^2x^2\sqrt{c^2x^2+1}+2\sqrt{c^2x^2+1}\right)\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^5,x)
```

```
[Out] c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arccsch(c*x)+1/32*(c^2*x^2+1)^(1/2)*(3*
arctanh(1/(c^2*x^2+1)^(1/2))*c^4*x^4-3*c^2*x^2*(c^2*x^2+1)^(1/2)+2*(c^2*x^2
+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^5/x^5))
```

Maxima [B] time = 1.01471, size = 198, normalized size = 2.68

$$\frac{1}{64}\,b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}+1\right)+1}}{c}-\frac{16\operatorname{arcsch}(cx)}{x^4}\right)-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c
^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1/(
c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1) +
 1))/c - 16*arccsch(c*x)/x^4) - 1/4*a/x^4
```

Fricas [A] time = 2.12589, size = 196, normalized size = 2.65

$$\frac{\left(3\,bc^4x^4-8\,b\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(3\,bc^3x^3-2\,bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-8\,a}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) - (3*b*c^3*x^3 - 2*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 8*a)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^5, x)
```

3.13 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{5x^5}+\frac{1}{25}bc^5\left(\frac{1}{c^2x^2}+1\right)^{5/2}-\frac{2}{15}bc^5\left(\frac{1}{c^2x^2}+1\right)^{3/2}+\frac{1}{5}bc^5\sqrt{\frac{1}{c^2x^2}+1}$$

```
[Out] (b*c^5*Sqrt[1 + 1/(c^2*x^2)])/5 - (2*b*c^5*(1 + 1/(c^2*x^2))^(3/2))/15 + (b
*c^5*(1 + 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsch[c*x])/(5*x^5)
```

Rubi [A] time = 0.051031, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6284, 266, 43}

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{5x^5}+\frac{1}{25}bc^5\left(\frac{1}{c^2x^2}+1\right)^{5/2}-\frac{2}{15}bc^5\left(\frac{1}{c^2x^2}+1\right)^{3/2}+\frac{1}{5}bc^5\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^6,x]
```

```
[Out] (b*c^5*Sqrt[1 + 1/(c^2*x^2)])/5 - (2*b*c^5*(1 + 1/(c^2*x^2))^(3/2))/15 + (b
*c^5*(1 + 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsch[c*x])/(5*x^5)
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{5x^5} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^7}\,dx}{5c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{5x^5} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{10c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{5x^5} + \frac{b\,\text{Subst}\left(\int \left(\frac{c^4}{\sqrt{1+\frac{x}{c^2}}} - 2c^4\sqrt{1+\frac{x}{c^2}} + c^4\left(1+\frac{x}{c^2}\right)^{3/2}\right)dx, x, \frac{1}{x^2}\right)}{10c}$$

$$= \frac{1}{5}bc^5\sqrt{1+\frac{1}{c^2x^2}} - \frac{2}{15}bc^5\left(1+\frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{25}bc^5\left(1+\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\text{csch}^{-1}(cx)}{5x^5}$$

Mathematica [A] time = 0.0610851, size = 69, normalized size = 0.87

$$-\frac{a}{5x^5} + b\left(-\frac{4c^3}{75x^2} + \frac{8c^5}{75} + \frac{c}{25x^4}\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^6,x]
```

```
[Out] -a/(5*x^5) + b*((8*c^5)/75 + c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(1 + c^2*x
^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(5*x^5)
```

Maple [A] time = 0.193, size = 83, normalized size = 1.1

$$c^5\left(-\frac{a}{5\,c^5x^5} + b\left(-\frac{\text{arccsch}\,(cx)}{5\,c^5x^5} + \frac{\left(c^2x^2+1\right)\left(8\,c^4x^4 - 4\,c^2x^2 + 3\right)}{75\,c^6x^6}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^6,x)
```

```
[Out] c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arccsch(c*x)+1/75*(c^2*x^2+1)*(8*c^4*x^
4-4*c^2*x^2+3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))
```

Maxima [A] time = 0.977892, size = 99, normalized size = 1.25

$$\frac{1}{75}\,b\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsch}(cx)}{x^5}\right)-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 1
5*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) - 1/5*a/x^5
```

Fricas [A] time = 2.12023, size = 197, normalized size = 2.49

$$-\frac{15\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(8\,bc^5x^5-4\,bc^3x^3+3\,bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+15\,a}{75\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(15*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*x
^5 - 4*b*c^3*x^3 + 3*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 15*a)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{acsch}(cx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^6, x)
```

3.14 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x^7}\,dx$

Optimal. Leaf size=98

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}+\frac{5bc^5\sqrt{\frac{1}{c^2x^2}+1}}{96x}-\frac{5bc^3\sqrt{\frac{1}{c^2x^2}+1}}{144x^3}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{36x^5}-\frac{5}{96}bc^6\operatorname{csch}^{-1}(cx)$$

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(144
*x^3) + (5*b*c^5*Sqrt[1 + 1/(c^2*x^2)])/(96*x) - (5*b*c^6*ArcCsch[c*x])/96
- (a + b*ArcCsch[c*x])/(6*x^6)
```

Rubi [A] time = 0.0636193, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6284, 335, 321, 215}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}+\frac{5bc^5\sqrt{\frac{1}{c^2x^2}+1}}{96x}-\frac{5bc^3\sqrt{\frac{1}{c^2x^2}+1}}{144x^3}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{36x^5}-\frac{5}{96}bc^6\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^7,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(144
*x^3) + (5*b*c^5*Sqrt[1 + 1/(c^2*x^2)])/(96*x) - (5*b*c^6*ArcCsch[c*x])/96
- (a + b*ArcCsch[c*x])/(6*x^6)
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^8}\,dx}{6c}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}+\frac{b\,\mathrm{Subst}\left(\int \frac{x^6}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{6c}\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}-\frac{1}{36}(5bc)\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}+\frac{1}{48}\left(5bc^3\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3}+\frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}-\frac{1}{96}\left(5bc^5\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\cdots}}\right.\\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3}+\frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x}-\frac{5}{96}bc^6\operatorname{csch}^{-1}(cx)-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}
\end{aligned}
$$

Mathematica [A] time = 0.0718543, size = 88, normalized size = 0.9

$$
-\frac{a}{6x^6}+b\left(-\frac{5c^3}{144x^3}+\frac{5c^5}{96x}+\frac{c}{36x^5}\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-\frac{5}{96}bc^6\sinh^{-1}\left(\frac{1}{cx}\right)-\frac{b\operatorname{csch}^{-1}(cx)}{6x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^7,x]
```

```
[Out] -a/(6*x^6) + b*(c/(36*x^5) - (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(1 +
c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(6*x^6) - (5*b*c^6*ArcSinh[1/(c*x)])
/96
```

Maple [A] time = 0.197, size = 139, normalized size = 1.4

$$c^6\left(-\frac{a}{6\,c^6x^6}+b\left(-\frac{\operatorname{arccsch}(cx)}{6\,c^6x^6}-\frac{1}{288\,c^7x^7}\sqrt{c^2x^2+1}\left(15\,\operatorname{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6-15\,c^4x^4\sqrt{c^2x^2+1}+10\,c^2x^2\sqrt{c^2x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^7,x)
```

```
[Out] c^6*(-1/6*a/c^6/x^6+b*(-1/6/c^6/x^6*arccsch(c*x)-1/288*(c^2*x^2+1)^(1/2)*(1
5*arctanh(1/(c^2*x^2+1)^(1/2))*c^6*x^6-15*c^4*x^4*(c^2*x^2+1)^(1/2)+10*c^2*
x^2*(c^2*x^2+1)^(1/2)-8*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^7/
x^7))
```

Maxima [B] time = 1.05192, size = 250, normalized size = 2.55

$$-\frac{1}{576}\,b\left(\frac{15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(15\,c^{12}x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\,c^{10}x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6x^6\left(\frac{1}{c^2x^2}+1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}+1\right)-1}}{c}+\frac{9}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 15*c^7*log(c*x*sqrt(
1/(c^2*x^2) + 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) + 1)^(5/2) - 40*c^10*x^
3*(1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^
```

```
2*x^2) + 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) + 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) + 1)
 - 1))/c + 96*arccsch(c*x)/x^6) - 1/6*a/x^6
```

Fricas [A] time = 2.17684, size = 225, normalized size = 2.3

$$-\frac{3\left(5\,bc^{6}x^{6}+16\,b\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)-\left(15\,bc^{5}x^{5}-10\,bc^{3}x^{3}+8\,bcx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+48\,a}{288\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/288*(3*(5*b*c^6*x^6 + 16*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)) - (15*b*c^5*x^5 - 10*b*c^3*x^3 + 8*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 48*a)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**7,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^7, x)
```

3.15 $\int x^3 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=105

$$\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{6c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{b^2x^2}{12c^2}-\frac{b^2\log(x)}{3c^4}$$

```
[Out] (b^2*x^2)/(12*c^2) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/(3*c^
3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*x]))/(6*c) + (x^4*(a + b
*ArcCsch[c*x])^2)/4 - (b^2*Log[x])/(3*c^4)
```

Rubi [A] time = 0.118674, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6286, 5452, 4185, 4184, 3475}

$$\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{6c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{b^2x^2}{12c^2}-\frac{b^2\log(x)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsch[c*x])^2,x]
```


```
[Out] (b^2*x^2)/(12*c^2) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/(3*c^
3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*x]))/(6*c) + (x^4*(a + b
*ArcCsch[c*x])^2)/4 - (b^2*Log[x])/(3*c^4)
```


Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\operatorname{csch}^4(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^4}\\ &= \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^2-\frac{b\,\text{Subst}\left(\int (a+bx)\operatorname{csch}^4(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2c^4}\\ &= \frac{b^2x^2}{12c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^2+\frac{b\,\text{Subst}\left(\int (a+bx)\text{c}\right.}{}\\ &= \frac{b^2x^2}{12c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\text{cs}\right.\\ &= \frac{b^2x^2}{12c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\text{cs}\right.\end{aligned}$$

Mathematica [A] time = 0.205312, size = 122, normalized size = 1.16

$$\frac{cx\left(3a^2c^3x^3+2ab\sqrt{\frac{1}{c^2x^2}+1}\left(c^2x^2-2\right)+b^2cx\right)+2bcx\operatorname{csch}^{-1}(cx)\left(3ac^3x^3+b\sqrt{\frac{1}{c^2x^2}+1}\left(c^2x^2-2\right)\right)+3b^2c^4x^4\operatorname{csch}^{-1}(cx)^2}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))
 + 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))*ArcCsch[c
*x] + 3*b^2*c^4*x^4*ArcCsch[c*x]^2 - 4*b^2*Log[x])/(12*c^4)
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int x^3 (a + b\mathrm{arccsch}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a*b + 1/288*(72*x^4*log(sqrt(c^2*x^2 + 1) +
 1)^2 + 1152*c^2*integrate(1/2*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 1152*c^2*integrate(1/2*x^5*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x)*log(c) + 576*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(
sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*c^2
*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqr
t(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 576*c^2*int
egrate(1/2*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2
 + 1) + 1), x) - 1152*c^2*integrate(1/2*x^5*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 1152
*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) - 1152*integrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
```

```
 24*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^
2 + 1) + 6)*log(c)^2/c^4 - 48*(3*c^2*x^2 - 2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c
^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*log(c)^2/c^4 + 144*(c^2*x^2 - 2*sqrt(
c^2*x^2 + 1) + 1)*log(c)^2/c^4 + 144*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1
))*log(c)^2/c^4 - 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2)
 - 12*sqrt(c^2*x^2 + 1) + 6)*log(c)*log(x)/c^4 + 288*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)*log(x)/c^4 + 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^
2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*log(c)*log(sqrt(c^2*x^2 + 1) +
 1)/c^4 - 288*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)*log(sqrt(c^2*x^2 +
 1) + 1)/c^4 + 4*(18*c^2*x^2 - 9*(c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) -
 96*sqrt(c^2*x^2 + 1) + 66*log(sqrt(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2
 + 1) - 1) + 18)*log(c)/c^4 + 4*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^
2 + 1)^(3/2) + 132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*
log(c)/c^4 - 144*(c^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) +
 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)*log(c)/c^4 + 144*(c^2*x^2 - 6*sqrt(c^
2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*log(c)/c^4 + 12*(6*c^2*x^2 -
 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*log(
sqrt(c^2*x^2 + 1) + 1)/c^4 + (6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 +
 1)^(3/2) + 132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)/c^4
 + 576*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*integrate(1/2*sqrt(c^2*x^
2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 576*integrate(1/2*x^3*log(x)^2/(sqrt(
c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*integrat
e(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^
2*x^2 + sqrt(c^2*x^2 + 1) + 1), x))*b^2
```

Fricas [B] time = 2.35634, size = 603, normalized size = 5.74

$$\frac{3\,b^2c^4x^4\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2+3\,a^2c^4x^4+6\,abc^4\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-6\,abc^4\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+b^2c^2x^2-4\,b}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 +
3*a^2*c^4*x^4 + 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1)
- 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + b^2*c^2*x^2
- 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 + (b^2*c^3*x^3 - 2*b^2*c*x)*s
```

```
qrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 2*(a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{acsch}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)^2 x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*x^3, x)
```

3.16 $\int x^2 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=122

$$-\frac{b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c}-\frac{2b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a\right.}{3c^3}$$

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcCsch[c*x]))/(3*c)
+ (x^3*(a + b*ArcCsch[c*x])^2)/3 - (2*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcC
sch[c*x]])/(3*c^3) - (b^2*PolyLog[2, -E^ArcCsch[c*x]])/(3*c^3) + (b^2*PolyL
og[2, E^ArcCsch[c*x]])/(3*c^3)
```

Rubi [A] time = 0.132321, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6286, 5452, 4185, 4182, 2279, 2391}

$$-\frac{b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c}-\frac{2b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a\right.}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcCsch[c*x]))/(3*c)
+ (x^3*(a + b*ArcCsch[c*x])^2)/3 - (2*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcC
sch[c*x]])/(3*c^3) - (b^2*PolyLog[2, -E^ArcCsch[c*x]])/(3*c^3) + (b^2*PolyL
og[2, E^ArcCsch[c*x]])/(3*c^3)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
```

```
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = -\frac{\text{Subst}\left(\int (a+bx)^2 \coth(x)\text{csch}^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^3}$$
$$= \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{(2b)\,\text{Subst}\left(\int (a+bx)\text{csch}^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{3c^3}$$
$$= \frac{b^2x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{b\,\text{Subst}\left(\int (a+bx)\text{cs}\right.}{3}$$
$$= \frac{b^2x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}$$
$$= \frac{b^2x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}$$
$$= \frac{b^2x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}$$

Mathematica [A] time = 1.36635, size = 211, normalized size = 1.73

$$b^2\text{PolyLog}\left(2,-e^{-\text{csch}^{-1}(cx)}\right) - b^2\text{PolyLog}\left(2,e^{-\text{csch}^{-1}(cx)}\right) + a^2c^3x^3 + abc^2x^2\sqrt{\frac{1}{c^2x^2}+1} - \frac{abcx\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}} + 2abc^3x^3\text{cs}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*c*x + a*b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2 + a^2*c^3*x^3 + b^2*c^2*Sqrt[1
 + 1/(c^2*x^2)]*x^2*ArcCsch[c*x] + 2*a*b*c^3*x^3*ArcCsch[c*x] + b^2*c^3*x^3
*ArcCsch[c*x]^2 - (a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*ArcSinh[c*x])/Sqrt[1 + c^2
*x^2] + b^2*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])] - b^2*ArcCsch[c*x]*Log[
1 + E^(-ArcCsch[c*x])] + b^2*PolyLog[2, -E^(-ArcCsch[c*x])] - b^2*PolyLog[2
, E^(-ArcCsch[c*x])])/(3*c^3)
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int x^2 \left(a+b\text{arccsch}\,(cx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3 + \frac{1}{6}\left(4x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)ab + \frac{1}{3}\left(x^3\log\left(\sqrt{c^2x^2+1}+1\right)^2 - 3\int -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*
x^2) + 1) - 1)/c^2)/c)*a*b + 1/3*(x^3*log(sqrt(c^2*x^2 + 1) + 1)^2 - 3*inte
grate(-1/3*(3*c^2*x^4*log(c)^2 + 3*x^2*log(c)^2 + 3*(c^2*x^4 + x^2)*log(x)^
2 + 6*(c^2*x^4*log(c) + x^2*log(c))*log(x) - 2*(3*c^2*x^4*log(c) + 3*x^2*lo
g(c) + 3*(c^2*x^4 + x^2)*log(x) + (c^2*x^4*(3*log(c) + 1) + 3*x^2*log(c) +
3*(c^2*x^4 + x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 3
*(c^2*x^4*log(c)^2 + x^2*log(c)^2 + (c^2*x^4 + x^2)*log(x)^2 + 2*(c^2*x^4*l
og(c) + x^2*log(c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/
2) + 1), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^2\operatorname{arcsch}(cx)^2 + 2\,abx^2\operatorname{arcsch}(cx) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arccsch(c*x)^2 + 2*a*b*x^2*arccsch(c*x) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{acsch}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*x^2, x)
```

3.17 $\int x \left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2 \,dx$

Optimal. Leaf size=54

$$\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/c + (x^2*(a + b*ArcCsch[c*
x])^2)/2 + (b^2*Log[x])/c^2
```

Rubi [A] time = 0.0760874, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6286, 5452, 4184, 3475}

$$\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/c + (x^2*(a + b*ArcCsch[c*
x])^2)/2 + (b^2*Log[x])/c^2
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx &= -\frac{\operatorname{Subst}\left(\int (a+bx)^2\coth(x)\operatorname{csch}^2(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2-\frac{b\operatorname{Subst}\left(\int (a+bx)\operatorname{csch}^2(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^2}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2-\frac{b^2\operatorname{Subst}\left(\int \coth(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^2}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.137106, size = 87, normalized size = 1.61

$$
\frac{acx\left(acx+2b\sqrt{\frac{1}{c^2x^2}+1}\right)+2bcx\operatorname{csch}^{-1}(cx)\left(acx+b\sqrt{\frac{1}{c^2x^2}+1}\right)+b^2c^2x^2\operatorname{csch}^{-1}(cx)^2+2b^2\log(cx)}{2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 + 1/(c^2*x^2
)] + a*c*x)*ArcCsch[c*x] + b^2*c^2*x^2*ArcCsch[c*x]^2 + 2*b^2*Log[c*x])/(2*
c^2)
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$
\int x\left(a+b\operatorname{arccsch}(cx)\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x*(a+b*arccsch(c*x))^2,x)
```

Maxima [A] time = 1.00935, size = 111, normalized size = 2.06

$$\frac{1}{2}\,b^2x^2\,\operatorname{arcsch}\left(cx\right)^2+\frac{1}{2}\,a^2x^2+\left(x^2\,\operatorname{arcsch}\left(cx\right)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)ab+\left(\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\operatorname{arcsch}\left(cx\right)}{c}+\frac{\log\left(x\right)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arccsch(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c^
2*x^2) + 1)/c)*a*b + (x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + log(x)/c^2)*
b^2
```

Fricas [B] time = 2.45099, size = 524, normalized size = 9.7

$$\frac{b^2c^2x^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2+a^2c^2x^2+2\,abc^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-2\,abc^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+2\,abcx\sqrt{\frac{c^2x^2+}{c^2x^2}}}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^2
*c^2*x^2 + 2*a*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*a
*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*a*b*c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 + b^2*c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)))/c^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x(a+b\,\mathrm{acsch}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcsch}\,(cx)+a)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*x, x)
```

3.18 $\int \left(a+b\,\text{csch}^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=68

$$\frac{2b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c}-\frac{2b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c}+x\left(a+b\,\text{csch}^{-1}(cx)\right)^2+\frac{4b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)(a+b\,\text{csch}}{c}$$

```
[Out] x*(a + b*ArcCsch[c*x])^2 + (4*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcCsch[c*x]
])/c + (2*b^2*PolyLog[2, -E^ArcCsch[c*x]])/c - (2*b^2*PolyLog[2, E^ArcCsch[
c*x]])/c
```

Rubi [A] time = 0.0676694, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6280, 5452, 4182, 2279, 2391}

$$\frac{2b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c}-\frac{2b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c}+x\left(a+b\,\text{csch}^{-1}(cx)\right)^2+\frac{4b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)(a+b\,\text{csch}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] x*(a + b*ArcCsch[c*x])^2 + (4*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcCsch[c*x]
])/c + (2*b^2*PolyLog[2, -E^ArcCsch[c*x]])/c - (2*b^2*PolyLog[2, E^ArcCsch[
c*x]])/c
```

Rule 6280

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \left(a+b\mathrm{csch}^{-1}(cx)\right)^2\,dx = -\frac{\mathrm{Subst}\left(\int (a+bx)^2\coth(x)\mathrm{csch}(x)\,dx,x,\mathrm{csch}^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\mathrm{csch}^{-1}(cx)\right)^2 - \frac{(2b)\,\mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(x)\,dx,x,\mathrm{csch}^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\mathrm{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\mathrm{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\mathrm{csch}^{-1}(cx)}\right)}{c} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \log\left(1-e\right.\right.}{c}$$

$$= x\left(a+b\mathrm{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\mathrm{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\mathrm{csch}^{-1}(cx)}\right)}{c} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,d\right.}{c}$$

$$= x\left(a+b\mathrm{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\mathrm{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\mathrm{csch}^{-1}(cx)}\right)}{c} + \frac{2b^2\mathrm{Li}_2\left(-e^{\mathrm{csch}^{-1}(cx)}\right)}{c} - \frac{2b}{}$$

Mathematica [A] time = 0.222216, size = 121, normalized size = 1.78

$$\frac{-2b^2\mathrm{PolyLog}\left(2,-e^{-\mathrm{csch}^{-1}(cx)}\right)+2b^2\mathrm{PolyLog}\left(2,e^{-\mathrm{csch}^{-1}(cx)}\right)+a^2cx+2abcx\mathrm{csch}^{-1}(cx)-2ab\log\left(\tanh\left(\frac{1}{2}\mathrm{csch}^{-1}(cx)\right)\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcCsch[c*x] + b^2*c*x*ArcCsch[c*x]^2 - 2*b^2*ArcCsch[
c*x]*Log[1 - E^(-ArcCsch[c*x])] + 2*b^2*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*
x])] - 2*a*b*Log[Tanh[ArcCsch[c*x]/2]] - 2*b^2*PolyLog[2, -E^(-ArcCsch[c*x]
)] + 2*b^2*PolyLog[2, E^(-ArcCsch[c*x])])/c
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccsch}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(x \log\left(\sqrt{c^2x^2+1}+1\right)^2 - \int -\frac{c^2x^2\log(c)^2 + \left(c^2x^2+1\right)\log(x)^2 + \log(c)^2 + 2\left(c^2x^2\log(c)+\log(c)\right)\log(x) - 2\left(c^2x^2\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] (x*log(sqrt(c^2*x^2 + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 +
 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x) - 2*(c^2*x^2*l
og(c) + (c^2*x^2 + 1)*log(x) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 + 1)*log(x)
 + log(c))*sqrt(c^2*x^2 + 1) + log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^
2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c)
)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x))*b^2 +
 a^2*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/
(c^2*x^2) + 1) - 1))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2 \operatorname{arcsch}(cx)^2 + 2\,ab\operatorname{arcsch}(cx) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsch}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2, x)
```

3.19 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=81

$$-b\,\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)+\frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,e^{2\mathrm{csch}^{-1}(cx)}\right)+\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{3b}-\log\left(1-e^{2\mathrm{csch}^{-1}(cx)}\right.$$

```
[Out] (a + b*ArcCsch[c*x])^3/(3*b) - (a + b*ArcCsch[c*x])^2*Log[1 - E^(2*ArcCsch[
c*x])] - b*(a + b*ArcCsch[c*x])*PolyLog[2, E^(2*ArcCsch[c*x])] + (b^2*PolyL
og[3, E^(2*ArcCsch[c*x])])/2
```

Rubi [A] time = 0.137782, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6286, 3716, 2190, 2531, 2282, 6589}

$$-b\,\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)+\frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,e^{2\mathrm{csch}^{-1}(cx)}\right)+\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{3b}-\log\left(1-e^{2\mathrm{csch}^{-1}(cx)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^3/(3*b) - (a + b*ArcCsch[c*x])^2*Log[1 - E^(2*ArcCsch[
c*x])] - b*(a + b*ArcCsch[c*x])*PolyLog[2, E^(2*ArcCsch[c*x])] + (b^2*PolyL
og[3, E^(2*ArcCsch[c*x])])/2
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = -\operatorname{Subst}\left(\int (a+bx)^2\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$
$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} + 2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$
$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + (2b)\operatorname{Subst}\left(\int (a+bx)\log\right.$$
$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{Li}_2\left(e^{2}\right.$$
$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{Li}_2\left(e^{2}\right.$$
$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{Li}_2\left(e^{2}\right.$$

Mathematica [A] time = 0.131099, size = 115, normalized size = 1.42

$$ab\left(\operatorname{PolyLog}\left(2, e^{-2\operatorname{csch}^{-1}(cx)}\right) - \operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx) + 2\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)\right) + b^2\left(-\operatorname{csch}^{-1}(cx)\operatorname{PolyLog}\left(2, e^{2\operatorname{csc}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x,x]
```

```
[Out] a^2*Log[c*x] + a*b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[
c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]) + b^2*(ArcCsch[c*x]^3/3 - ArcCs
ch[c*x]^2*Log[1 - E^(2*ArcCsch[c*x])] - ArcCsch[c*x]*PolyLog[2, E^(2*ArcCsc
h[c*x])] + PolyLog[3, E^(2*ArcCsch[c*x])]/2)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{arccsch}(cx)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2\log(x)\log\left(\sqrt{c^2x^2+1}+1\right)^2+a^2\log(x)-\int -\frac{b^2\log(c)^2+\left(b^2c^2\log(c)^2-2\,abc^2\log(c)\right)x^2-2\,ab\log(c)+\left(b^2c^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2 + a^2*log(x) - integrate(-(b^2*log(
c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2 - 2*a*b*log(c) + (b^2*c^2*
x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)
*log(x) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x
^2 + b^2)*log(x) + sqrt(c^2*x^2 + 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*
log(c) - a*b + (2*b^2*c^2*x^2 + b^2)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1) +
sqrt(c^2*x^2 + 1)*(b^2*log(c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2
 - 2*a*b*log(c) + (b^2*c^2*x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c
^2)*x^2 + b^2*log(c) - a*b)*log(x)))/(c^2*x^3 + (c^2*x^3 + x)*sqrt(c^2*x^2
+ 1) + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{arcsch}(cx)^2+2\,ab\operatorname{arcsch}(cx)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{acsch}(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x, x)
```

3.20 $\int \frac{\left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}-\frac{2b^2}{x}$$

```
[Out] (-2*b^2)/x + 2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]) - (a + b*ArcC
sch[c*x])^2/x
```

Rubi [A] time = 0.069934, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6286, 3296, 2637}

$$2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}-\frac{2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^2,x]
```

```
[Out] (-2*b^2)/x + 2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]) - (a + b*ArcC
sch[c*x])^2/x
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx = -\left(c\,\mathrm{Subst}\left(\int (a+bx)^2\cosh(x)\,dx, x, \mathrm{csch}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x} + (2bc)\,\mathrm{Subst}\left(\int (a+bx)\sinh(x)\,dx, x, \mathrm{csch}^{-1}(cx)\right)$$
$$= 2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\,\mathrm{csch}^{-1}(cx)\right) - \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x} - \left(2b^2c\right)\mathrm{Subst}\left(\int \cosh(x)\,dx, x, \mathrm{c}\right.$$
$$= -\frac{2b^2}{x} + 2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\,\mathrm{csch}^{-1}(cx)\right) - \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x}$$

Mathematica [A] time = 0.162027, size = 70, normalized size = 1.43

$$-\frac{a^2 - 2abcx\sqrt{\frac{1}{c^2x^2}+1} + 2b\,\mathrm{csch}^{-1}(cx)\left(a - bcx\sqrt{\frac{1}{c^2x^2}+1}\right) + b^2\mathrm{csch}^{-1}(cx)^2 + 2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^2,x]
```

```
[Out] -((a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 2*b*(a - b*c*Sqrt[1 + 1/
(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*ArcCsch[c*x]^2)/x)
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{arccsch}\,(cx)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^2,x)
```

Maxima [A] time = 1.01186, size = 105, normalized size = 2.14

$$2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)ab+2\left(c\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx)-\frac{1}{x}\right)b^2-\frac{b^2\operatorname{arcsch}(cx)^2}{x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) +
1)*arccsch(c*x) - 1/x)*b^2 - b^2*arccsch(c*x)^2/x - a^2/x
```

Fricas [B] time = 2.05663, size = 296, normalized size = 6.04

$$\frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2-a^2-2\,b^2+2\left(b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-ab\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b^2*log((c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - a*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{acsch}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x}-\frac{1}{2}abc^2\operatorname{csch}^{-1}(cx)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2x^2}-\frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2-\frac{b^2}{4x^2}$$

```
[Out] -b^2/(4*x^2) - (a*b*c^2*ArcCsch[c*x])/2 - (b^2*c^2*ArcCsch[c*x]^2)/4 + (b*c
*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*x) - (a + b*ArcCsch[c*x])^2
/(2*x^2)
```

Rubi [A] time = 0.0824147, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6286, 5446, 3310}

$$\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x}-\frac{1}{2}abc^2\operatorname{csch}^{-1}(cx)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2x^2}-\frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2-\frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^3,x]
```

```
[Out] -b^2/(4*x^2) - (a*b*c^2*ArcCsch[c*x])/2 - (b^2*c^2*ArcCsch[c*x]^2)/4 + (b*c
*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*x) - (a + b*ArcCsch[c*x])^2
/(2*x^2)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx = -\left(c^2\,\text{Subst}\left(\int (a+bx)^2\cosh(x)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2} + \left(bc^2\right)\text{Subst}\left(\int (a+bx)\sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= -\frac{b^2}{4x^2} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{2x} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2} - \frac{1}{2}\left(bc^2\right)\text{Subst}\left(\int (a+bx)\right.$$

$$= -\frac{b^2}{4x^2} - \frac{1}{2}abc^2\text{csch}^{-1}(cx) - \frac{1}{4}b^2c^2\text{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{2x} - \frac{\left(a+b\text{cs\right.}}{2}$$

Mathematica [A] time = 0.136994, size = 100, normalized size = 1.15

$$-\frac{2a^2 - 2abcx\sqrt{\frac{1}{c^2x^2}+1} + 2abc^2x^2\sinh^{-1}\left(\frac{1}{cx}\right) - 2b\text{csch}^{-1}(cx)\left(bcx\sqrt{\frac{1}{c^2x^2}+1} - 2a\right) + b^2\left(c^2x^2+2\right)\text{csch}^{-1}(cx)^2 + b^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^3,x]
```

```
[Out] -(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 2*b*(-2*a + b*c*Sqrt[1 +
1/(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*(2 + c^2*x^2)*ArcCsch[c*x]^2 + 2*a*b*c^2
*x^2*ArcSinh[1/(c*x)])/(4*x^2)
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{arccsch}\,(cx)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}ab\left(\frac{\frac{2c^4x\sqrt{\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}+1\right)-1}-c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)+c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c}-\frac{4\operatorname{arcsch}(cx)}{x^2}\right)-\frac{1}{2}b^2\left(\frac{\log\left(\sqrt{c^2x^2+1}+1\right)^2}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c
^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) -
 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^2 +
2*integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2
*x^2*log(c) + log(c))*log(x) - (2*c^2*x^2*log(c) + 2*(c^2*x^2 + 1)*log(x) +
 (c^2*x^2*(2*log(c) - 1) + 2*(c^2*x^2 + 1)*log(x) + 2*log(c))*sqrt(c^2*x^2
+ 1) + 2*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^2*log(c)^2 + (c^2*x^2
+ 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*x^2
 + 1))/(c^2*x^5 + x^3 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a^2/x
^2
```

Fricas [B] time = 2.11415, size = 350, normalized size = 4.02

$$\frac{2abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-\left(b^2c^2x^2+2b^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2-2a^2-b^2-2\left(abc^2x^2-b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2ab\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b^2*c^2*x^2 + 2*b^2)*log((c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 - 2*(a*b*c^2*x
^2 - b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=100

$$-\frac{4}{9}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\text{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\text{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{3x^3}+\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

```
[Out] (-2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a +
 b*ArcCsch[c*x]))/9 + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(9
*x^2) - (a + b*ArcCsch[c*x])^2/(3*x^3)
```

Rubi [A] time = 0.10826, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6286, 5446, 3310, 3296, 2637}

$$-\frac{4}{9}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\text{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\text{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{3x^3}+\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^4,x]
```

```
[Out] (-2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a +
 b*ArcCsch[c*x]))/9 + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(9
*x^2) - (a + b*ArcCsch[c*x])^2/(3*x^3)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx &= -\left(c^3\operatorname{Subst}\left(\int (a+bx)^2\cosh(x)\sinh^2(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bc^3\right)\operatorname{Subst}\left(\int (a+bx)\sinh^3(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\\
&= -\frac{2b^2}{27x^3}+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}-\frac{1}{9}\left(4bc^3\right)\operatorname{Subst}\left(\int (a+\right.\\
&= -\frac{2b^2}{27x^3}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\operatorname{csc}\right.}{3}\\
&= -\frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2}-
\end{aligned}$$

Mathematica [A] time = 0.183789, size = 106, normalized size = 1.06

$$\frac{-9a^2+6abcx\sqrt{\frac{1}{c^2x^2}+1}\left(1-2c^2x^2\right)-6b\operatorname{csch}^{-1}(cx)\left(3a+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(2c^2x^2-1\right)\right)+2b^2\left(6c^2x^2-1\right)-9b^2\operatorname{csch}^{-1}(cx)^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^4,x]
```

```
[Out] (-9*a^2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c
^2*x^2) - 6*b*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2))*ArcCsch[
c*x] - 9*b^2*ArcCsch[c*x]^2)/(27*x^3)
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \frac{(a + b\mathrm{arccsch}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{9}\,ab\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\mathrm{arcsch}\,(cx)}{x^3}\right) - \frac{1}{3}\,b^2\left(\frac{\log\left(\sqrt{c^2x^2+1}+1\right)^2}{x^3} + 3\int -\frac{3\,c^2x^2\log(c)^2 + 3\left(c^2x^2+1\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*
arccsch(c*x)/x^3) - 1/3*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^3 + 3*integrate
(-1/3*(3*c^2*x^2*log(c)^2 + 3*(c^2*x^2 + 1)*log(x)^2 + 3*log(c)^2 + 6*(c^2*
x^2*log(c) + log(c))*log(x) - 2*(3*c^2*x^2*log(c) + 3*(c^2*x^2 + 1)*log(x)
+ (c^2*x^2*(3*log(c) - 1) + 3*(c^2*x^2 + 1)*log(x) + 3*log(c))*sqrt(c^2*x^2
 + 1) + 3*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 3*(c^2*x^2*log(c)^2 + (c^2*x
^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*
x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1)), x)) - 1/3*a^
2/x^3
```

Fricas [B] time = 2.20302, size = 385, normalized size = 3.85

$$\frac{12\,b^2c^2x^2 - 9\,b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 9\,a^2 - 2\,b^2 - 6\left(3\,ab + \left(2\,b^2c^3x^3 - b^2cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - 6\left(2\,abc^3x^3 - \right.}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x))^2 - 9*a^2 - 2*b^2 - 6*(3*a*b + (2*b^2*c^3*x^3 - b^2*c*x)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 6*(2
*a*b*c^3*x^3 - a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\textbf{csch}^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=132

$$-\frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{16x}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3}+\frac{3}{16}abc^4\text{csch}^{-1}(cx)-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4}+\frac{3b^2c^2}{32x^2}$$

```
[Out] -b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsch[c*x])/16 + (3*b^2
*c^4*ArcCsch[c*x]^2)/32 + (b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/
(8*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(16*x) - (a
+ b*ArcCsch[c*x])^2/(4*x^4)
```

Rubi [A] time = 0.116934, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6286, 5446, 3310}

$$-\frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{16x}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3}+\frac{3}{16}abc^4\text{csch}^{-1}(cx)-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4}+\frac{3b^2c^2}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^5,x]
```

```
[Out] -b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsch[c*x])/16 + (3*b^2
*c^4*ArcCsch[c*x]^2)/32 + (b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/
(8*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(16*x) - (a
+ b*ArcCsch[c*x])^2/(4*x^4)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
```

```
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = -\left(c^4 \operatorname{Subst}\left(\int (a+bx)^2\cosh(x)\sinh^3(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right)\operatorname{Subst}\left(\int (a+bx)\sinh^4(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$

$$= -\frac{b^2}{32x^4} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{8x^3} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x^4} - \frac{1}{8}\left(3bc^4\right)\operatorname{Subst}\left(\int (a+b$$

$$= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16x} - \frac{(a$$

$$= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\operatorname{csch}^{-1}(cx) + \frac{3}{32}b^2c^4\operatorname{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx\right.}{8x^3}$$

Mathematica [A] time = 0.173547, size = 147, normalized size = 1.11

$$\frac{-8a^2 - 6abc^3x^3\sqrt{\frac{1}{c^2x^2}+1} + 4abcx\sqrt{\frac{1}{c^2x^2}+1} + 6abc^4x^4\sinh^{-1}\left(\frac{1}{cx}\right) - 2b\operatorname{csch}^{-1}(cx)\left(8a+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(3c^2x^2-2\right)\right) + 3}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 - b^2 + 4*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^3
*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3
*c^2*x^2))*ArcCsch[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^2 + 6*a*b*c^4*x
^4*ArcSinh[1/(c*x)])/(32*x^4)
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{arccsch}\,(cx))^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^5,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{32}\,ab\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}+1\right)+1}}{c}-\frac{16\,\mathrm{arcsch}\,(cx)}{x^4}\right)-\frac{1}{4}\,b^2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/
(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1)
 + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^4
 + 4*integrate(-1/2*(2*c^2*x^2*log(c)^2 + 2*(c^2*x^2 + 1)*log(x)^2 + 2*log(
c)^2 + 4*(c^2*x^2*log(c) + log(c))*log(x) - (4*c^2*x^2*log(c) + 4*(c^2*x^2
+ 1)*log(x) + (c^2*x^2*(4*log(c) - 1) + 4*(c^2*x^2 + 1)*log(x) + 4*log(c))*
sqrt(c^2*x^2 + 1) + 4*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(c^2*x^2*log(c
)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x
))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1)),
x)) - 1/4*a^2/x^4
```

Fricas [A] time = 2.22867, size = 432, normalized size = 3.27

$$\frac{3\,b^2c^2x^2 + \left(3\,b^2c^4x^4 - 8\,b^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 8\,a^2 - b^2 + 2\left(3\,abc^4x^4 - 8\,ab - \left(3\,b^2c^3x^3 - 2\,b^2cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2}{c}}}{c}\right.}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x))^2 - 8*a^2 - b^2 + 2*(3*a*b*c^4*x^4 - 8*a*b - (3*b^2*c
^3*x^3 - 2*b^2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 +
1)/(c^2*x^2)))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\operatorname{acsch}\left(cx\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\operatorname{arcsch}\left(cx\right) + a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^5, x)
```

3.24 $\int x^3 \left(a + b\,\mathbf{csch}^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=195

$$\frac{b^3 \text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right)}{2c^4} + \frac{b^2 x^2 \left(a + b\text{csch}^{-1}(cx)\right)}{4c^2} + \frac{b^2 \log\left(1 - e^{2\text{csch}^{-1}(cx)}\right)\left(a + b\text{csch}^{-1}(cx)\right)}{c^4} + \frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a + bc\right.}{4c}$$

```
[Out] (b^3*Sqrt[1 + 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsch[c*x]))/(4*c
^2) - (b*(a + b*ArcCsch[c*x])^2)/(2*c^4) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a +
b*ArcCsch[c*x])^2)/(2*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*
x])^2)/(4*c) + (x^4*(a + b*ArcCsch[c*x])^3)/4 + (b^2*(a + b*ArcCsch[c*x])*L
og[1 - E^(2*ArcCsch[c*x])])/c^4 + (b^3*PolyLog[2, E^(2*ArcCsch[c*x])])/(2*c
^4)
```

Rubi [A] time = 0.232396, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6286, 5452, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{b^3 \text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right)}{2c^4} + \frac{b^2 x^2 \left(a + b\text{csch}^{-1}(cx)\right)}{4c^2} + \frac{b^2 \log\left(1 - e^{2\text{csch}^{-1}(cx)}\right)\left(a + b\text{csch}^{-1}(cx)\right)}{c^4} + \frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a + bc\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[1 + 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsch[c*x]))/(4*c
^2) - (b*(a + b*ArcCsch[c*x])^2)/(2*c^4) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a +
b*ArcCsch[c*x])^2)/(2*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*
x])^2)/(4*c) + (x^4*(a + b*ArcCsch[c*x])^3)/4 + (b^2*(a + b*ArcCsch[c*x])*L
og[1 - E^(2*ArcCsch[c*x])])/c^4 + (b^3*PolyLog[2, E^(2*ArcCsch[c*x])])/(2*c
^4)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\text{csch}^4(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^4}$$
$$=\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\text{csch}^4(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{4c^4}$$
$$=\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+$$
$$=\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)}{4c}$$
$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}\right.}{2c^3}$$
$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}\right.}{2c^3}$$
$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}\right.}{2c^3}$$
$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}\right.}{2c^3}$$

Mathematica [A] time = 0.974611, size = 271, normalized size = 1.39

$$\frac{-2b^3\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)+b\text{csch}^{-1}(cx)\left(cx\left(3a^2c^3x^3+2ab\sqrt{\frac{1}{c^2x^2}+1}\left(c^2x^2-2\right)+b^2cx\right)+4b^2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (-2*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x + a*b^2
*c^2*x^2 + a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^4
*x^4 + b*(2 - 2*c*Sqrt[1 + 1/(c^2*x^2)]*x + c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3))
*ArcCsch[c*x]^2 + b^3*c^4*x^4*ArcCsch[c*x]^3 + b*ArcCsch[c*x]*(c*x*(b^2*c*x
 + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2)) + 4*b^2*Log[
1 - E^(-2*ArcCsch[c*x])]) + 4*a*b^2*Log[1/(c*x)] - 2*b^3*PolyLog[2, E^(-2*A
rcCsch[c*x])])/(4*c^4)
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int x^{3} \left(a + b \operatorname{arccsch}\left(c x\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log(sqrt(c^2*x^2 + 1) + 1)^3 - 12*b^3*c^2*integrate(1/4*x^5*log
(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c
)^2 + 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 1/4*a^3*x^4
 - 12*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x^2 +
1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integ
rate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^2
*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2
```

*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^2*integrate(1/4*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*a*b^2*c^2*integrate(1/4*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 4*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 12*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 4*b^3*c^2*integrate(1/4*x^5*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3*c^2*integrate(1/4*x^5*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 12*b^3*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 3*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 12*b^3*integrate(1/4*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 12*b^3*integrate(1/4*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*integrate(1/4*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*integrate(1/4*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*integrate(1/4*x^3*log(sqrt(c^2*x^2

```
 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1),
 x)*log(c) + 24*a*b^2*integrate(1/4*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
 c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*integrate(1/4*x^3*l
og(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) + 1/4*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) +
 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a^2*b - 4*b^3*integrate(1/4*sqr
t(c^2*x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x) + 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)^2*lo
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x) - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqr
t(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
 1) + 1), x) - 4*b^3*integrate(1/4*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2
 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3*integrate(1/4*x^3*log(x)^2*
log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*
x^2 + 1) + 1), x) - 12*b^3*integrate(1/4*x^3*log(x)*log(sqrt(c^2*x^2 + 1) +
 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 1
2*a*b^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2
*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*integrate(1/4*sqrt(c
^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*integrate(1/4*sqrt(c^2*
x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*integrate(1/4*x^3*log(x)^2/(sqr
t(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*in
tegrate(1/4*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*integrate(1/4*x^3*log(s
qrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2
 + 1) + 1), x) + 1/12*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2
) - 12*sqrt(c^2*x^2 + 1) + 6)*b^3*log(c)^3/c^4 + 1/6*(3*c^2*x^2 - 2*(c^2*x^
2 + 1)^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*b^3*log(c)^3/c
^4 - 1/2*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*b^3*log(c)^3/c^4 - 1/2*b^3*(2*
sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*log(c)^3/c^4 + 1/4*(6*c^2*x^2 - 3*(c^
2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*b^3*log(c)
^2*log(x)/c^4 - 3/2*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*b^3*log(c)^2*log(x)
/c^4 - 1/4*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt
(c^2*x^2 + 1) + 6)*b^3*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^4 + 3/2*(c^2*x
^2 - 2*sqrt(c^2*x^2 + 1) + 1)*b^3*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^4 -
 1/4*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x
^2 + 1) + 6)*a*b^2*log(c)^2/c^4 - 1/2*(3*c^2*x^2 - 2*(c^2*x^2 + 1)^(3/2) +
6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*a*b^2*log(c)^2/c^4 + 3/2*(c^2
*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*a*b^2*log(c)^2/c^4 - 1/48*(18*c^2*x^2 - 9*(
c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) - 96*sqrt(c^2*x^2 + 1) + 66*log(sqr
t(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2 + 1) - 1) + 18)*b^3*log(c)^2/c^4
 - 1/48*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 + 1)^(3/2) + 132*sqrt(c
^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*b^3*log(c)^2/c^4 + 3/4*(c
^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log(sqrt(c^2*
```

```
x^2 + 1) - 1) + 1)*b^3*log(c)^2/c^4 - 3/4*(c^2*x^2 - 6*sqrt(c^2*x^2 + 1) +
6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*b^3*log(c)^2/c^4 + 3/2*a*b^2*(2*sqrt(c^2*
x^2 + 1) - log(c^2*x^2 + 1))*log(c)^2/c^4 - 1/2*(6*c^2*x^2 - 3*(c^2*x^2 + 1
)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*a*b^2*log(c)*log(x)
/c^4 + 3*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*a*b^2*log(c)*log(x)/c^4 + 1/2*
(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 +
1) + 6)*a*b^2*log(c)*log(sqrt(c^2*x^2 + 1) + 1)/c^4 - 3*(c^2*x^2 - 2*sqrt(c
^2*x^2 + 1) + 1)*a*b^2*log(c)*log(sqrt(c^2*x^2 + 1) + 1)/c^4 + 1/24*(18*c^2
*x^2 - 9*(c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) - 96*sqrt(c^2*x^2 + 1) +
66*log(sqrt(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2 + 1) - 1) + 18)*a*b^2*l
og(c)/c^4 + 1/24*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 + 1)^(3/2) +
132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*a*b^2*log(c)/c^
4 - 3/2*(c^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log
(sqrt(c^2*x^2 + 1) - 1) + 1)*a*b^2*log(c)/c^4 + 3/2*(c^2*x^2 - 6*sqrt(c^2*x
^2 + 1) + 6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*a*b^2*log(c)/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\,\text{arcsch}\,(cx)^3 + 3\,ab^2x^3\,\text{arcsch}\,(cx)^2 + 3\,a^2bx^3\,\text{arcsch}\,(cx) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arccsch(c*x)^3 + 3*a*b^2*x^3*arccsch(c*x)^2 + 3*a^2*b*x^3*
arccsch(c*x) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,(a + b\,\text{acsch}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)^3 x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*x^3, x)
```

3.25 $\int x^2 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)^3 \,dx$

Optimal. Leaf size=194

$$-\frac{b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)\left(a+b\,\text{csch}^{-1}(cx)\right)}{c^3}+\frac{b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)\left(a+b\,\text{csch}^{-1}(cx)\right)}{c^3}+\frac{b^3\text{PolyLog}\left(3,-e^{\text{csch}^{-1}}\right.}{c^3}$$

```
[Out] (b^2*x*(a + b*ArcCsch[c*x]))/c^2 + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcC
sch[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsch[c*x])^3)/3 - (b*(a + b*ArcCsch[c*x
])^2*ArcTanh[E^ArcCsch[c*x]])/c^3 + (b^3*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c^
3 - (b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c^3 + (b^2*(a +
b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c^3 + (b^3*PolyLog[3, -E^ArcCsc
h[c*x]])/c^3 - (b^3*PolyLog[3, E^ArcCsch[c*x]])/c^3
```

Rubi [A] time = 0.208091, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6286, 5452, 4186, 3770, 4182, 2531, 2282, 6589}

$$-\frac{b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)\left(a+b\,\text{csch}^{-1}(cx)\right)}{c^3}+\frac{b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)\left(a+b\,\text{csch}^{-1}(cx)\right)}{c^3}+\frac{b^3\text{PolyLog}\left(3,-e^{\text{csch}^{-1}}\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcCsch[c*x]))/c^2 + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcC
sch[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsch[c*x])^3)/3 - (b*(a + b*ArcCsch[c*x
])^2*ArcTanh[E^ArcCsch[c*x]])/c^3 + (b^3*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c^
3 - (b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c^3 + (b^2*(a +
b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c^3 + (b^3*PolyLog[3, -E^ArcCsc
h[c*x]])/c^3 - (b^3*PolyLog[3, E^ArcCsch[c*x]])/c^3
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=-\frac{\operatorname{Subst}\left(\int(a+bx)^3\coth(x)\operatorname{csch}^3(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^3}$$
$$=\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{b\operatorname{Subst}\left(\int(a+bx)^2\operatorname{csch}^3(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^3}$$
$$=\frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3+\frac{b\,S}{}$$
$$=\frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{b\,(}{}$$
$$=\frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{b\,(}{}$$
$$=\frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{b\,(}{}$$
$$=\frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{b\,(}{}$$

Mathematica [B] time = 7.48386, size = 548, normalized size = 2.82

$$\frac{ab^2\left(2c^3x^3\left(-\frac{4\operatorname{PolyLog}\left(2,e^{-\operatorname{csch}^{-1}(cx)}\right)}{c^3x^3}+4\operatorname{csch}^{-1}(cx)^2+2\cosh\left(2\operatorname{csch}^{-1}(cx)\right)-\frac{3\operatorname{csch}^{-1}(cx)\log\left(1-e^{-\operatorname{csch}^{-1}(cx)}\right)}{cx}+\frac{3\operatorname{csch}^{-1}(cx)\log\left(e^{-\operatorname{csch}}\right.}{cx}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3*A
rcCsch[c*x] - (a^2*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3) +
(a*b^2*(8*PolyLog[2, -E^(-ArcCsch[c*x])] + 2*c^3*x^3*(-2 + 4*ArcCsch[c*x]^2
 + 2*Cosh[2*ArcCsch[c*x]] - (3*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])])/(c*
```

```
x) + (3*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*x])])/(c*x) - (4*PolyLog[2, E^(-
ArcCsch[c*x])])/(c^3*x^3) + 2*ArcCsch[c*x]*Sinh[2*ArcCsch[c*x]] + ArcCsch[c
*x]*Log[1 - E^(-ArcCsch[c*x])]*Sinh[3*ArcCsch[c*x]] - ArcCsch[c*x]*Log[1 +
E^(-ArcCsch[c*x])]*Sinh[3*ArcCsch[c*x]])))/(8*c^3) + (b^3*(24*ArcCsch[c*x]*
Coth[ArcCsch[c*x]/2] - 4*ArcCsch[c*x]^3*Coth[ArcCsch[c*x]/2] + 6*ArcCsch[c*
x]^2*Csch[ArcCsch[c*x]/2]^2 + (ArcCsch[c*x]^3*Csch[ArcCsch[c*x]/2]^4)/(c*x)
 + 24*ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] - 24*ArcCsch[c*x]^2*Log[1 +
 E^(-ArcCsch[c*x])] - 48*Log[Tanh[ArcCsch[c*x]/2]] + 48*ArcCsch[c*x]*PolyLo
g[2, -E^(-ArcCsch[c*x])] - 48*ArcCsch[c*x]*PolyLog[2, E^(-ArcCsch[c*x])] +
48*PolyLog[3, -E^(-ArcCsch[c*x])] - 48*PolyLog[3, E^(-ArcCsch[c*x])] + 6*Ar
cCsch[c*x]^2*Sech[ArcCsch[c*x]/2]^2 + 16*c^3*x^3*ArcCsch[c*x]^3*Sinh[ArcCsc
h[c*x]/2]^4 - 24*ArcCsch[c*x]*Tanh[ArcCsch[c*x]/2] + 4*ArcCsch[c*x]^3*Tanh[
ArcCsch[c*x]/2]))/(48*c^3)
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int x^2 (a + b\operatorname{arccsch}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log(sqrt(c^2*x^2 + 1) + 1)^3 + 1/3*a^3*x^3 + 1/4*(4*x^3*arccsch
(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1
/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b - i
ntegrate(((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b^3*c^2*x^4 + b^
3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*((b^3*c^2*log(c
) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + (3*(b^3*c^2*log(c
) - a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)
```

```
*log(x) + ((b^3*c^2*(3*log(c) + 1) - 3*a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b
^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2
*x^2 + 1) + 1)^2 + 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*lo
g(c)^2 - 2*a*b^2*log(c))*x^2)*log(x) - 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*l
og(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*
log(x)^2 + 2*((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*
log(x) + ((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a
*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*log(x)^2 + 2*((b^3*c^2*log(c) -
a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(s
qrt(c^2*x^2 + 1) + 1) + ((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b
^3*c^2*x^4 + b^3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*
((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + 3*
((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(
c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2 \operatorname{arcsch}(cx)^3 + 3\,ab^2x^2 \operatorname{arcsch}(cx)^2 + 3\,a^2bx^2 \operatorname{arcsch}(cx) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arccsch(c*x)^3 + 3*a*b^2*x^2*arccsch(c*x)^2 + 3*a^2*b*x^2*
arccsch(c*x) + a^3*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b \operatorname{acsch}(cx)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))**3, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcsch}\,(cx)+a)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*x^2, x)
```

3.26 $\int x \left(a+b\text{csch}^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=117

$$-\frac{3b^3\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2c^2}-\frac{3b^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{3bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{3b\left(a+\right.}{}$$

```
[Out] (3*b*(a + b*ArcCsch[c*x])^2)/(2*c^2) + (3*b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*
ArcCsch[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsch[c*x])^3)/2 - (3*b^2*(a + b*Arc
Csch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/c^2 - (3*b^3*PolyLog[2, E^(2*ArcCsc
h[c*x])])/(2*c^2)
```

Rubi [A] time = 0.159845, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6286, 5452, 4184, 3716, 2190, 2279, 2391}

$$-\frac{3b^3\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2c^2}-\frac{3b^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{3bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{3b\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsch[c*x])^3,x]
```


```
[Out] (3*b*(a + b*ArcCsch[c*x])^2)/(2*c^2) + (3*b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*
ArcCsch[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsch[c*x])^3)/2 - (3*b^2*(a + b*Arc
Csch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/c^2 - (3*b^3*PolyLog[2, E^(2*ArcCsc
h[c*x])])/(2*c^2)
```


Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
```

```
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int(a+bx)^3\coth(x)\text{csch}^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^2}$$

$$=\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{(3b)\,\text{Subst}\left(\int(a+bx)^2\text{csch}^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{2c^2}$$

$$=\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{\left(3b^2\right)\text{Subst}\left(\int(a+bx)\,\text{coth}\right.}{c^2}$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{(6b}{}$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3b^2}{}$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3b^2}{}$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3b^2}{}$$

Mathematica [A] time = 0.45747, size = 171, normalized size = 1.46

$$\frac{3b^3\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)+a\left(acx\left(acx+3b\sqrt{\frac{1}{c^2x^2}+1}\right)-6b^2\log\left(\frac{1}{cx}\right)\right)+3b^2\text{csch}^{-1}(cx)^2\left(ac^2x^2+b\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right.\right.}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (3*b^2*(a*c^2*x^2 + b*(-1 + c*Sqrt[1 + 1/(c^2*x^2)]*x))*ArcCsch[c*x]^2 + b^
3*c^2*x^2*ArcCsch[c*x]^3 + 3*b*ArcCsch[c*x]*(a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2
)] + a*c*x) - 2*b^2*Log[1 - E^(-2*ArcCsch[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 +
1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + 3*b^3*PolyLog[2, E^(-2*ArcCsc
h[c*x])])/(2*c^2)
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int x\left(a+b\text{arccsch}\,(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x*(a+b*arccsch(c*x))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arccsch(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsch(c*x) + x*sqrt
(1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + lo
g(x)/c^2)*a*b^2 - 1/4*(24*c^2*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*c^2*integrate(1
/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 2*x^2*log(sqrt(c^2*x^2 + 1) + 1)^3 + 24
*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 48*c^2*integrate(1/2*sqrt
(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integrate(1/2*sq
rt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2
 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integrate(1/2*x^3*l
og(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*l
og(c) - 48*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^
2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*i
ntegrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 8*c^2*integrate(1/2*sqrt(c^2*
x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
 1) + 1), x) - 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2*log(sqrt
(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
 + 1), x) + 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*
x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x) + 8*c^2*integrate(1/2*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*c^2*integrate(1/2*x^3*log(x)^2*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x) + 24*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*c^2*
integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*
sqrt(c^2*x^2 + 1)*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*
```

```
x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*integrate(1/2*x
*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
 48*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*
integrate(1/2*x*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x)*log(c) - 48*integrate(1/2*x*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)
 + 24*integrate(1/2*x*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 2*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)^3/c^2 + 2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*log
(c)^3/c^2 + 2*(log(c^2*x^2 + 1) - 2*log(sqrt(c^2*x^2 + 1) + 1))*log(c)^3/c^
2 + 6*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(x)/c^2 - 6*(c^2*x^2
- 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^2 + 4*log(
c)^3*log(sqrt(c^2*x^2 + 1) + 1)/c^2 - 6*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)
^2/c^2 + 4*log(c)*log(sqrt(c^2*x^2 + 1) + 1)^3/c^2 - 3*(c^2*x^2 - 4*sqrt(c^
2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)
*log(c)^2/c^2 + 3*(c^2*x^2 - 6*sqrt(c^2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1)
+ 1) + 1)*log(c)^2/c^2 + 8*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^3/(sqrt
(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integrate
(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
+ 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*sqrt
(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 8*integrate(1/2*x*log(x)^3/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integra
te(1/2*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*x*log(x)*log(sqrt(c^
2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
+ 1), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x\,\mathrm{arcsch}\,(cx)^3+3\,ab^2x\,\mathrm{arcsch}\,(cx)^2+3\,a^2bx\,\mathrm{arcsch}\,(cx)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arccsch(c*x)^3 + 3*a*b^2*x*arccsch(c*x)^2 + 3*a^2*b*x*arccsc
h(c*x) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{acsch}\left(cx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{arcsch}\left(cx\right)+a\right)^{3}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*x, x)
```

3.27 $\int \left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=120

$$\frac{6b^2\mathrm{PolyLog}\left(2,-e^{\mathrm{csch}^{-1}(cx)}\right)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{c}-\frac{6b^2\mathrm{PolyLog}\left(2,e^{\mathrm{csch}^{-1}(cx)}\right)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{c}-\frac{6b^3\mathrm{PolyLog}\left(3,-e^{\mathrm{csch}}\right.}{c}$$

```
[Out] x*(a + b*ArcCsch[c*x])^3 + (6*b*(a + b*ArcCsch[c*x])^2*ArcTanh[E^ArcCsch[c*
x]])/c + (6*b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c - (6*b^
2*(a + b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c - (6*b^3*PolyLog[3, -E
^ArcCsch[c*x]])/c + (6*b^3*PolyLog[3, E^ArcCsch[c*x]])/c
```

Rubi [A] time = 0.117491, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6280, 5452, 4182, 2531, 2282, 6589}

$$\frac{6b^2\mathrm{PolyLog}\left(2,-e^{\mathrm{csch}^{-1}(cx)}\right)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{c}-\frac{6b^2\mathrm{PolyLog}\left(2,e^{\mathrm{csch}^{-1}(cx)}\right)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{c}-\frac{6b^3\mathrm{PolyLog}\left(3,-e^{\mathrm{csch}}\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3,x]
```



```
[Out] x*(a + b*ArcCsch[c*x])^3 + (6*b*(a + b*ArcCsch[c*x])^2*ArcTanh[E^ArcCsch[c*
x]])/c + (6*b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c - (6*b^
2*(a + b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c - (6*b^3*PolyLog[3, -E
^ArcCsch[c*x]])/c + (6*b^3*PolyLog[3, E^ArcCsch[c*x]])/c
```


Rule 6280

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^3 dx = -\frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\text{csch}(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c}$$
$$= x\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c}$$
$$= x\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{\left(6b^2\right)\text{Subst}\left(\int (a+bx)\right.}{}$$
$$= x\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{L}}{c}$$
$$= x\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{L}}{c}$$
$$= x\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{L}}{c}$$

Mathematica [B] time = 0.323685, size = 246, normalized size = 2.05

$$\frac{3ab^2\left(-2\text{PolyLog}\left(2,-e^{-\text{csch}^{-1}(cx)}\right)+2\text{PolyLog}\left(2,e^{-\text{csch}^{-1}(cx)}\right)+\text{csch}^{-1}(cx)\left(cx\text{csch}^{-1}(cx)-2\log\left(1-e^{-\text{csch}^{-1}(cx)}\right)+2\right.\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcCsch[c*x] + (3*a^2*b*Log[c*x*(1 + Sqrt[(1 + c^2*x^2)/(
c^2*x^2)])])/c + (3*a*b^2*(ArcCsch[c*x]*(c*x*ArcCsch[c*x] - 2*Log[1 - E^(-A
rcCsch[c*x])] + 2*Log[1 + E^(-ArcCsch[c*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c
*x])] + 2*PolyLog[2, E^(-ArcCsch[c*x])]))/c + (b^3*(c*x*ArcCsch[c*x]^3 - 3*
ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] + 3*ArcCsch[c*x]^2*Log[1 + E^(-Ar
cCsch[c*x])] - 6*ArcCsch[c*x]*PolyLog[2, -E^(-ArcCsch[c*x])] + 6*ArcCsch[c*
x]*PolyLog[2, E^(-ArcCsch[c*x])] - 6*PolyLog[3, -E^(-ArcCsch[c*x])] + 6*Pol
yLog[3, E^(-ArcCsch[c*x])]))/c
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (a+b\text{arccsch}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*x + 3/2*(2*c*x*arccsch(c*x) + log(
sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*a^2*b/c - inte
grate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^
3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c
^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log
(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(b^3*
log(c) - a*b^2 + (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^
3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c)
 + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x) - 3*(b^3*log(c)^2 -
2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2
 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2
)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2
*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*
c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 +
1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 +
 (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c)
 + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/
(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3\,\mathrm{arcsch}\,(cx)^3 + 3\,ab^2\,\mathrm{arcsch}\,(cx)^2 + 3\,a^2b\,\mathrm{arcsch}\,(cx) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x)
 + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{acsch}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arcsch}\,(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3, x)
```

3.28 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x}\,dx$

Optimal. Leaf size=110

$$\frac{3}{2}b^2\mathrm{PolyLog}\left(3,e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)-\frac{3}{2}b\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)^2-\frac{3}{4}b^3\mathrm{PolyLog}\left(4,e^{2\mathrm{cs}}\right.$$

```
[Out] (a + b*ArcCsch[c*x])^4/(4*b) - (a + b*ArcCsch[c*x])^3*Log[1 - E^(2*ArcCsch[
c*x])] - (3*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2 + (3
*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])])/2 - (3*b^3*PolyLo
g[4, E^(2*ArcCsch[c*x])])/4
```

Rubi [A] time = 0.153647, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6286, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}b^2\mathrm{PolyLog}\left(3,e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)-\frac{3}{2}b\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)^2-\frac{3}{4}b^3\mathrm{PolyLog}\left(4,e^{2\mathrm{cs}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^4/(4*b) - (a + b*ArcCsch[c*x])^3*Log[1 - E^(2*ArcCsch[
c*x])] - (3*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2 + (3
*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])])/2 - (3*b^3*PolyLo
g[4, E^(2*ArcCsch[c*x])])/4
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx = -\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$
$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} + 2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)$$
$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + (3b)\,\text{Subst}\left(\int (a+bx)^2\,\text{lo}\right.$$
$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{Li}_2$$
$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{Li}_2$$
$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{Li}_2$$
$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{Li}_2$$

Mathematica [A] time = 0.215751, size = 198, normalized size = 1.8

$$\frac{1}{4}\left(6a^2b\left(\text{PolyLog}\left(2, e^{-2\text{csch}^{-1}(cx)}\right) - \text{csch}^{-1}(cx)\left(\text{csch}^{-1}(cx) + 2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right)\right) + 2ab^2\left(-6\text{csch}^{-1}(cx)\text{PolyLog}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x,x]
```

```
[Out] (4*a^3*Log[c*x] + 6*a^2*b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*A
rcCsch[c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]) + 2*a*b^2*(2*ArcCsch[c*x
]^2*(ArcCsch[c*x] - 3*Log[1 - E^(2*ArcCsch[c*x])]) - 6*ArcCsch[c*x]*PolyLog
[2, E^(2*ArcCsch[c*x])] + 3*PolyLog[3, E^(2*ArcCsch[c*x])]) + b^3*(ArcCsch[
c*x]^4 - 4*ArcCsch[c*x]^3*Log[1 - E^(2*ArcCsch[c*x])] - 6*ArcCsch[c*x]^2*Po
lyLog[2, E^(2*ArcCsch[c*x])] + 6*ArcCsch[c*x]*PolyLog[3, E^(2*ArcCsch[c*x])
] - 3*PolyLog[4, E^(2*ArcCsch[c*x])]))/4
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \frac{(a+b\,\text{arccsch}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*log(x) - integrate((b^3*log(c
)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b
^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2*log(c))*x^2 + 3*(b^3*l
og(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)
- a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + s
qrt(c^2*x^2 + 1)*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (
2*b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^
2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b
*c^2)*x^2)*log(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log
(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2
+ 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*l
og(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) +
 a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2
 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x
^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2
*log(c))*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*lo
g(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a
*b^2*c^2*log(c) + a^2*b*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^3 + (c^
2*x^3 + x)*sqrt(c^2*x^2 + 1) + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\operatorname{arcsch}(cx)^3 + 3\,ab^2\operatorname{arcsch}(cx)^2 + 3\,a^2b\operatorname{arcsch}(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x, x)
```

3.29 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=78

$$-\frac{6b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}+3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x}+6b^3c\sqrt{\frac{1}{c^2x^2}+1}$$

```
[Out] 6*b^3*c*Sqrt[1 + 1/(c^2*x^2)] - (6*b^2*(a + b*ArcCsch[c*x]))/x + 3*b*c*Sqrt
[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2 - (a + b*ArcCsch[c*x])^3/x
```

Rubi [A] time = 0.0989242, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6286, 3296, 2638}

$$-\frac{6b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}+3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x}+6b^3c\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^2,x]
```

```
[Out] 6*b^3*c*Sqrt[1 + 1/(c^2*x^2)] - (6*b^2*(a + b*ArcCsch[c*x]))/x + 3*b*c*Sqrt
[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2 - (a + b*ArcCsch[c*x])^3/x
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx = -\left(c\operatorname{Subst}\left(\int (a+bx)^3\cosh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} + (3bc)\operatorname{Subst}\left(\int (a+bx)^2\sinh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$
$$= 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} - \left(6b^2c\right)\operatorname{Subst}\left(\int (a+bx)\cosh(\right.$$
$$= -\frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} + \left(6b^3c\right.$$
$$= 6b^3c\sqrt{1+\frac{1}{c^2x^2}} - \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}\right.}{x}$$

Mathematica [A] time = 0.23174, size = 132, normalized size = 1.69

$$-\frac{3b\operatorname{csch}^{-1}(cx)\left(a^2-2abcx\sqrt{\frac{1}{c^2x^2}+1}+2b^2\right)-3a^2bcx\sqrt{\frac{1}{c^2x^2}+1}+a^3+3b^2\operatorname{csch}^{-1}(cx)^2\left(a-bcx\sqrt{\frac{1}{c^2x^2}+1}\right)+6ab^2-6b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^2,x]
```

```
[Out] -((a^3 + 6*a*b^2 - 3*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 + 1/(
c^2*x^2)]*x + 3*b*(a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c
*x] + 3*b^2*(a - b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x]^2 + b^3*ArcCsch[
c*x]^3)/x)
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{arccsch}(cx)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^2,x)
```

Maxima [A] time = 0.992252, size = 194, normalized size = 2.49

$$-\frac{b^3\,\mathrm{arcsch}\,(cx)^3}{x}+3\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\mathrm{arcsch}\,(cx)}{x}\right)a^2b+6\left(c\sqrt{\frac{1}{c^2x^2}+1}\,\mathrm{arcsch}\,(cx)-\frac{1}{x}\right)ab^2+3\left(c\sqrt{\frac{1}{c^2x^2}+1}\,\mathrm{arcsch}\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arccsch(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a^2*b
+ 6*(c*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2*x
^2) + 1)*arccsch(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) + 1) - 2*arccsch(c*x)/x)*b^3
 - 3*a*b^2*arccsch(c*x)^2/x - a^3/x
```

Fricas [B] time = 2.31187, size = 477, normalized size = 6.12

$$-\frac{b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-3\left(a^2b+2\,b^3\right)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+a^3+6\,ab^2-3\left(b^3cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-ab^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2-3\left(2\,ab^2cx\sqrt{\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2*b
^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^2 - 3*(2*a*b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - a^2*b - 2*b^3)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{acsch}\,(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^2, x)
```

3.30 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$-\frac{3b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{4x^2}+\frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{4x}-\frac{1}{4}c^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{2x^2}+\frac{3b^3c\sqrt{\frac{1}{c^2x^2}}}{8x}$$

```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsch[c*x])/8 - (3*b^2
*(a + b*ArcCsch[c*x]))/(4*x^2) + (3*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsc
h[c*x])^2)/(4*x) - (c^2*(a + b*ArcCsch[c*x])^3)/4 - (a + b*ArcCsch[c*x])^3/
(2*x^2)
```

Rubi [A] time = 0.106324, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6286, 5446, 3311, 32, 2635, 8}

$$-\frac{3b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{4x^2}+\frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{4x}-\frac{1}{4}c^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{2x^2}+\frac{3b^3c\sqrt{\frac{1}{c^2x^2}}}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^3,x]
```


```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsch[c*x])/8 - (3*b^2
*(a + b*ArcCsch[c*x]))/(4*x^2) + (3*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsc
h[c*x])^2)/(4*x) - (c^2*(a + b*ArcCsch[c*x])^3)/4 - (a + b*ArcCsch[c*x])^3/
(2*x^2)
```


Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
```

```
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx = -\left(c^2\,\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2} + \frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$
$$= -\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2} - \frac{1}{4}\left(3bc^2\right.$$
$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\left(a+b\text{csc}\right.$$
$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\text{csch}^{-1}(cx) - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx\right.}{4x}$$

Mathematica [A] time = 0.310988, size = 182, normalized size = 1.48

$$-\frac{3bc^2x^2\left(2a^2+b^2\right)\sinh^{-1}\left(\frac{1}{cx}\right) + 6b\text{csch}^{-1}(cx)\left(2a^2-2abcx\sqrt{\frac{1}{c^2x^2}+1}+b^2\right) - 6a^2bcx\sqrt{\frac{1}{c^2x^2}+1} + 4a^3 + 6b^2\text{csch}^{-1}(cx)^2}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^3,x]
```

```
[Out] -(4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1 + 1/
(c^2*x^2)]*x + 6*b*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[
c*x] + 6*b^2*(-(b*c*Sqrt[1 + 1/(c^2*x^2)]*x) + a*(2 + c^2*x^2))*ArcCsch[c*x
]^2 + 2*b^3*(2 + c^2*x^2)*ArcCsch[c*x]^3 + 3*b*(2*a^2 + b^2)*c^2*x^2*ArcSin
h[1/(c*x)])/(8*x^2)
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{arccsch}\,(cx)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) -
 c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
 - 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^2 -
 1/2*a^3/x^2 - integrate(1/2*(2*b^3*log(c)^3 - 6*a*b^2*log(c)^2 + 2*(b^3*c^
2*x^2 + b^3)*log(x)^3 + 2*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 6
*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(2*b^
3*log(c) - 2*a*b^2 + 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 +
b^3)*log(x) + sqrt(c^2*x^2 + 1)*(2*b^3*log(c) - 2*a*b^2 + (b^3*c^2*(2*log(c
) - 1) - 2*a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2
 + 1) + 1)^2 + 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b
^2*c^2*log(c))*x^2)*log(x) - 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*lo
g(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*lo
g(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2
*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2
+ b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)
*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(b^3*log(c)^3 -
3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b
^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)
*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a
*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^5 + x^3 + (c^2*x^5
+ x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [B] time = 2.46831, size = 576, normalized size = 4.68

$$-\frac{2\left(b^{3}c^{2}x^{2}+2\,b^{3}\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)^{3}-3\left(2\,a^{2}b+b^{3}\right)cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+4\,a^{3}+6\,ab^{2}+6\left(ab^{2}c^{2}x^{2}-b^{3}cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+2\,ab^{2}\right)\log}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(b^3*c^2*x^2 + 2*b^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x))^3 - 3*(2*a^2*b + b^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 4*a^3 + 6*a
*b^2 + 6*(a*b^2*c^2*x^2 - b^3*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b^2)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(4*a*b^2*c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acsch}\,(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcsch}\,(cx) + a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^3, x)
```

3.31 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=166

$$\frac{4b^2c^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3x}-\frac{2b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{9x^3}-\frac{2}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx\right.}{3x^2}$$

```
[Out] (-14*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)])/9 + (2*b^3*c^3*(1 + 1/(c^2*x^2))^(3/2))
/27 - (2*b^2*(a + b*ArcCsch[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsch[c*x]
))/(3*x) - (2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/3 + (b*c*
Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(3*x^2) - (a + b*ArcCsch[c*x]
)^3/(3*x^3)
```

Rubi [A] time = 0.170505, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6286, 5446, 3311, 3296, 2638, 2633}

$$\frac{4b^2c^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3x}-\frac{2b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{9x^3}-\frac{2}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx\right.}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^4,x]
```

```
[Out] (-14*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)])/9 + (2*b^3*c^3*(1 + 1/(c^2*x^2))^(3/2))
/27 - (2*b^2*(a + b*ArcCsch[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsch[c*x]
))/(3*x) - (2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/3 + (b*c*
Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(3*x^2) - (a + b*ArcCsch[c*x]
)^3/(3*x^3)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = -\left(c^3\,\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh^2(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right)\text{Subst}\left(\int (a+bx)^2\sinh^3(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$
$$= -\frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3} - \frac{1}{3}\left(2bc^3\right.$$
$$= -\frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} - \frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}\right.}{3x^2}$$
$$= -\frac{2}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2\left(a+b\operatorname{csch}^{-1}(\right.}{3x}$$
$$= -\frac{14}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2\left(a+b\operatorname{csch}^{-1}\right.}{3x}$$

Mathematica [A] time = 0.300845, size = 200, normalized size = 1.2

$$\frac{3b\operatorname{csch}^{-1}(cx)\left(-9a^2+6abcx\sqrt{\frac{1}{c^2x^2}+1}\left(1-2c^2x^2\right)+2b^2\left(6c^2x^2-1\right)\right)+9a^2bcx\sqrt{\frac{1}{c^2x^2}+1}\left(1-2c^2x^2\right)-9a^3+6ab^2\left(6c^2x\right.}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 + 2*b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 20*c^2*x^2) + 9*a^2*b*c*Sqrt
[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 6*a*b^2*(-1 + 6*c^2*x^2) + 3*b*(-9*a^
2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c^2*x^2
))*ArcCsch[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2)
)*ArcCsch[c*x]^2 - 9*b^3*ArcCsch[c*x]^3)/(27*x^3)
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{arccsch}(cx)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c -
3*arccsch(c*x)/x^3) - 1/3*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^3 - 1/3*a^3/x^
3 - integrate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)
^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2
+ (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (3*b^3*log(c) - 3*a*b^2 + 3*
(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 3*(b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*
x^2 + 1)*(3*b^3*log(c) - 3*a*b^2 + (b^3*c^2*(3*log(c) - 1) - 3*a*b^2*c^2)*x
^2 + 3*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*l
og(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log
(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*lo
g(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2
*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^
2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^
3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 +
 1))*log(sqrt(c^2*x^2 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c
^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*
(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*l
og(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log
(x))*sqrt(c^2*x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))
, x)
```

Fricas [B] time = 2.56577, size = 640, normalized size = 3.86

$$\frac{36\,ab^2c^2x^2 - 9\,b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 9\,a^3 - 6\,ab^2 - 9\left(3\,ab^2 + \left(2\,b^3c^3x^3 - b^3cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 + 3\left(12\,b^3c^2 \cdots\right.}{27\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^3 - 9*a^3 - 6*a*b^2 - 9*(3*a*b^2 + (2*b^3*c^3*x^3 - b^3*c*x)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^
2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b - 2*b^3 - 6*(2*a*b^2*c^3*x^3 - a*b^2*c*x)*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 - (9*a^2*b + 2*b^3)*c*x)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^4, x)
```

3.32 $\int \frac{\left(a+b\mathbf{csch}^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal. Leaf size=204

$$\frac{9b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^2}-\frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^4}-\frac{9bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{32x}+\frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16x^3}$$

```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(128*x^3) - (45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2
)])/(256*x) + (45*b^3*c^4*ArcCsch[c*x])/256 - (3*b^2*(a + b*ArcCsch[c*x]))/
(32*x^4) + (9*b^2*c^2*(a + b*ArcCsch[c*x]))/(32*x^2) + (3*b*c*Sqrt[1 + 1/(c
^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*
(a + b*ArcCsch[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsch[c*x])^3)/32 - (a + b
*ArcCsch[c*x])^3/(4*x^4)
```

Rubi [A] time = 0.181361, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6286, 5446, 3311, 32, 2635, 8}

$$\frac{9b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^2}-\frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^4}-\frac{9bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{32x}+\frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^5,x]
```


```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(128*x^3) - (45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2
)])/(256*x) + (45*b^3*c^4*ArcCsch[c*x])/256 - (3*b^2*(a + b*ArcCsch[c*x]))/
(32*x^4) + (9*b^2*c^2*(a + b*ArcCsch[c*x]))/(32*x^2) + (3*b*c*Sqrt[1 + 1/(c
^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*
(a + b*ArcCsch[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsch[c*x])^3)/32 - (a + b
*ArcCsch[c*x])^3/(4*x^4)
```


Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx = -\left(c^4\,\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\sinh^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= -\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{16x^3} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4} - \frac{1}{16}\left(9bc\right.$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{cs}\right.}{16x^3}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^2} + \frac{3b}{}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} + \frac{45}{256}b^3c^4\text{csch}^{-1}(cx) - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a\right.}{}$$

Mathematica [A] time = 0.336004, size = 277, normalized size = 1.36

$$\frac{9bc^4x^4\left(8a^2+5b^2\right)\sinh^{-1}\left(\frac{1}{cx}\right) - 24b\text{csch}^{-1}(cx)\left(8a^2+2abcx\sqrt{\frac{1}{c^2x^2}+1}\left(3c^2x^2-2\right)+b^2\left(1-3c^2x^2\right)\right) - 72a^2bc^3x^3\sqrt{\frac{1}{c^2x^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 - 24*a*b^2 + 48*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 +
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3
 - 45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 24*b*(8*a^2 + b^2*(1 - 3*c^2*x^2)
 + 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3*c^2*x^2))*ArcCsch[c*x] + 24*b^2*
(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(2 - 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcCsch[
c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^3 + 9*b*(8*a^2 + 5*b^2)*c^4*x^
4*ArcSinh[1/(c*x)])/(256*x^4)
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{arccsch}\left(cx\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^5,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(
1/(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt
(1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) +
1) + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^
4 - 1/4*a^3/x^4 - integrate(1/4*(4*b^3*log(c)^3 - 12*a*b^2*log(c)^2 + 4*(b^
3*c^2*x^2 + b^3)*log(x)^3 + 4*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2
 + 12*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*
(4*b^3*log(c) - 4*a*b^2 + 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 4*(b^3*c^2*x
^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(4*b^3*log(c) - 4*a*b^2 + (b^3*c^2*(4*
log(c) - 1) - 4*a*b^2*c^2)*x^2 + 4*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^
2*x^2 + 1) + 1)^2 + 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 -
 2*a*b^2*c^2*log(c))*x^2)*log(x) - 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3
*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*
(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c
)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c
^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^
2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 4*(b^3*log(
c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3
- 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b
^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^
2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c
^2*x^7 + x^5)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 2.47163, size = 747, normalized size = 3.66

$$\frac{72\,ab^2c^2x^2+8\left(3\,b^3c^4x^4-8\,b^3\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-64\,a^3-24\,ab^2+24\left(3\,ab^2c^4x^4-8\,ab^2-\left(3\,b^3c^3x^3-2\,b^3cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x))^3 - 64*a^3 - 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8
*a*b^2 - (3*b^3*c^3*x^3 - 2*b^3*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(3*(8*a^2*b + 5*b^3)*c^4*
x^4 + 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 - 16*(3*a*b^2*c^3*x^3 - 2*a*b^2*c*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 - 2*(8*a^2*b + b^3)*c*x)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\text{acsch}\,(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcsch}\,(cx)+a)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\,\mathrm{csch}^{-1}(cx)}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x}{a+b\,\text{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[x/(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.0151715, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{a+b\,\text{csch}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][x/(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \frac{x}{a+b\,\text{csch}^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\text{csch}^{-1}(cx)}\,dx$$

Mathematica [A] time = 2.87465, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\,\text{csch}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[x/(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{x}{a + b\mathrm{arccsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccsch(c*x)),x)
```

```
[Out] int(x/(a+b*arccsch(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\,\mathrm{arcsch}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccsch(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{b\,\mathrm{arcsch}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(x/(b*arccsch(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b\,\mathrm{acsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x/(a + b*acsch(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\,\mathrm{arcsch}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arccsch(c*x) + a), x)
```

3.34 $\int \frac{1}{a+b\,\mathbf{csch}^{-1}(cx)}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{a+b\,\text{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])^(-1), x]
```

Rubi [A] time = 0.0058766, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{a+b\,\text{csch}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\,\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\text{csch}^{-1}(cx)}\,dx$$

Mathematica [A] time = 2.69809, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\text{csch}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])^(-1),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])^(-1), x]
```

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int (a + b\mathrm{arccsch}\,(cx))^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/(a+b*arccsch(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\mathrm{arcsch}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccsch(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b\,\mathrm{arcsch}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccsch(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\,\mathrm{acsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(a + b*acsch(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arcsch}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccsch(c*x) + a), x)
```

3.35 $\int \frac{1}{x\left(a+b\,\mathbf{csch}^{-1}(cx)\right)} \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*ArcCsch[c*x])), x]
```

Rubi [A] time = 0.0261483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCsch[c*x])),x]

[Out] Defer[Int][1/(x*(a + b*ArcCsch[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)} \, dx=\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)} \, dx$$

Mathematica [A] time = 0.285686, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCsch[c*x])),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCsch[c*x])), x]
```

Maple [A] time = 0.21, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{arccsch}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x/(a+b*arccsch(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arcsch}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{bx\operatorname{arcsch}\left(cx\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*arccsch(c*x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(a+b\,\mathrm{acsch}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(x*(a + b*acsch(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arcsch}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x), x)
```

3.36 $\int \frac{1}{x^2\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=46

$$\frac{c\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}$$

```
[Out] -((c*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/b) + (c*Sinh[a/b]*SinhInte
gral[a/b + ArcCsch[c*x]])/b
```

Rubi [A] time = 0.104262, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6286, 3303, 3298, 3301}

$$\frac{c\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -((c*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/b) + (c*Sinh[a/b]*SinhInte
gral[a/b + ArcCsch[c*x]])/b
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)$$
$$= -\left(\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right) + \left(c\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{a}\right.$$
$$= -\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}$$

Mathematica [A] time = 0.0723812, size = 44, normalized size = 0.96

$$-\frac{c\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -((c*(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcCsch[c*x]]))/b)
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\operatorname{arccsch}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x^2/(a+b*arccsch(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcsch}(cx) + a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^2\operatorname{arcsch}(cx) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*arccsch(c*x) + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,(a + b\operatorname{acsch}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*acsch(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcsch}(cx) + a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

3.37 $\int \frac{1}{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=63

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}$$

```
[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*Cos
h[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]])/(2*b)
```

Rubi [A] time = 0.140565, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6286, 5448, 12, 3303, 3298, 3301}

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*Cos
h[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]])/(2*b)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx &= -\left(c^2\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)\\
&= -\left(c^2\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{2}c^2\operatorname{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{2}\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)+\frac{1}{2}\left(c^2\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Sub}\\
&= \frac{c^2\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0715245, size = 56, normalized size = 0.89

$$\frac{c^2\left(\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)-\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^2*(CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*
SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]]))/(2*b)
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b\,\mathrm{arccsch}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x^3/(a+b*arccsch(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arcsch}\left(cx\right) + a)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{bx^{3}\,\mathrm{arcsch}\left(cx\right) + ax^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^3*arccsch(c*x) + a*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \operatorname{acsch}(cx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(x**3*(a + b*acsch(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

3.38 $\int \frac{1}{x^4\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=117

$$\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}+\frac{c^3\sinh}{}$$

```
[Out] (c^3*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]
*CoshIntegral[(3*a)/b + 3*ArcCsch[c*x]])/(4*b) - (c^3*Sinh[a/b]*SinhIntegra
l[a/b + ArcCsch[c*x]])/(4*b) + (c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcCsch[c*x]])/(4*b)
```

Rubi [A] time = 0.240291, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6286, 5448, 3303, 3298, 3301}

$$\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}+\frac{c^3\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^3*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]
*CoshIntegral[(3*a)/b + 3*ArcCsch[c*x]])/(4*b) - (c^3*Sinh[a/b]*SinhIntegra
l[a/b + ArcCsch[c*x]])/(4*b) + (c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcCsch[c*x]])/(4*b)
```

Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx &= -\left(c^3\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\operatorname{csch}^{-1}(cx)\right)\right)\\
&= -\left(c^3\operatorname{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)\right)\\
&= \frac{1}{4}c^3\operatorname{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\operatorname{csch}^{-1}(cx)\right)-\frac{1}{4}c^3\operatorname{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\operatorname{csch}^{-1}(cx)\right)\\
&= \frac{1}{4}\left(c^3\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\operatorname{csch}^{-1}(cx)\right)-\frac{1}{4}\left(c^3\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \cdots\right.\\
&= \frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\sinh\left(\frac{a}{b}\right)\operatorname{Sh}\cdots}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.156946, size = 91, normalized size = 0.78

$$-\frac{c^3\left(-\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\cdots\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -(c^3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]]) + Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCsch[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcCsch[c*x]]
 - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCsch[c*x])]))/(4*b)
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a + b\,\mathrm{arccsch}\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x^4/(a+b*arccsch(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arcsch}\left(cx\right) + a)x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{bx^{4}\,\mathrm{arcsch}\left(cx\right) + ax^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*arccsch(c*x) + a*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a + b \operatorname{acsch}\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(x**4*(a + b*acsch(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arcsch}\left(cx\right) + a)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

3.39 $\int (dx)^m \left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3 \,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Rubi [A] time = 0.0242095, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^3 \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^3 \,dx = \int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^3 \,dx$$

Mathematica [A] time = 4.83209, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^3 \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Maple [A] time = 0.183, size = 0, normalized size = 0.

$$\int (dx)^m (a + b\operatorname{arccsch}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3 \operatorname{arcsch}(cx)^3 + 3\,ab^2 \operatorname{arcsch}(cx)^2 + 3\,a^2 b \operatorname{arcsch}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*(d*x)^m, x)
```

3.40 $\int (dx)^m \left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Rubi [A] time = 0.0251134, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 3.19842, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\,\text{csch}^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int (dx)^m (a + b\mathrm{arccsch}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2 \,\mathrm{arcsch}\,(cx)^2 + 2\,ab\,\mathrm{arcsch}\,(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)*(d*x)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b\,\mathrm{acsch}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsch(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcsch}(cx) + a)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*(d*x)^m, x)
```

3.41 $\int (dx)^m \left(a+b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=67

$$\frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},-\frac{1}{c^2x^2}\right)}{cm(m+1)}+\frac{(dx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometri
c2F1[1/2, -m/2, 1 - m/2, -(1/(c^2*x^2))])/(c*m*(1 + m))
```

Rubi [A] time = 0.0451583, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6284, 339, 364}

$$\frac{(dx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometri
c2F1[1/2, -m/2, 1 - m/2, -(1/(c^2*x^2))])/(c*m*(1 + m))
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)}+\frac{(bd)\int\frac{(dx)^{-1+m}}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c(1+m)}$$

$$=\frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)}-\frac{\left(b\left(\frac{1}{x}\right)^m(dx)^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c(1+m)}$$

$$=\frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

Mathematica [A] time = 0.230991, size = 81, normalized size = 1.21

$$\frac{x(dx)^m\left(\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)}{\sqrt{c^2x^2+1}}+(m+1)\left(a+b\text{csch}^{-1}(cx)\right)\right)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(d*x)^m*((1 + m)*(a + b*ArcCsch[c*x]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/Sqrt[1 + c^2*x^2]))/
(1 + m)^2
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int (dx)^m\left(a+b\text{arccsch}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsch(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\,\text{arcsch}\,(cx)+a)\,(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\,(a+b\,\text{acsch}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*acsch(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arcsch}\,(cx)+a)\,(dx)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(d*x)^m, x)
```

3.42 $\int \frac{(dx)^m}{a+b\textbf{csch}^{-1}(cx)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\text{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.027665, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.361548, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.186, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b\mathrm{arccsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsch(c*x)),x)
```

```
[Out] int((d*x)^m/(a+b*arccsch(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{arcsch}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b\,\mathrm{arcsch}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arccsch(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b\,\mathrm{acsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsch(c*x)),x)
```

```
[Out] Integral((d*x)**m/(a + b*acsch(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{arcsch}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\,\text{csch}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Rubi [A] time = 0.0271329, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.718919, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Maple [A] time = 0.196, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b\operatorname{arccsch}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2 d^m x^3 + d^m x\right)\sqrt{c^2x^2+1}\,x^m + \left(c^2 d^m x^3 + d^m x\right)x^m}{\left(b^2c^2\log(c) - abc^2\right)x^2 + b^2\log(c) - ab + \left(b^2c^2x^2 + b^2\right)\log(x) - \left(b^2c^2x^2 + \sqrt{c^2x^2+1}\,b^2 + b^2\right)\log\left(\sqrt{c^2x^2+1}+1\right) + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*d^m*x^3 + d^m*x)*sqrt(c^2*x^2 + 1)*x^m + (c^2*d^m*x^3 + d^m*x)*x^m)/
((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^2)*lo
g(x) - (b^2*c^2*x^2 + sqrt(c^2*x^2 + 1)*b^2 + b^2)*log(sqrt(c^2*x^2 + 1) +
1) + sqrt(c^2*x^2 + 1)*(b^2*log(c) + b^2*log(x) - a*b)) - integrate(-((c^2*
d^m*(m + 3)*x^2 + d^m*(m + 1))*(c^2*x^2 + 1)*x^m + (c^4*d^m*(m + 2)*x^4 + c
^2*d^m*(3*m + 5)*x^2 + 2*d^m*(m + 1))*sqrt(c^2*x^2 + 1)*x^m + (c^4*d^m*(m +
 1)*x^4 + 2*c^2*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c) - a*b*
c^4)*x^4 + 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) + (c^2*x^2 + 1)*(b
^2*log(c) + b^2*log(x) - a*b) - a*b + (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*l
og(x) - (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + (c^2*x^2 + 1)*b^2 + b^2 + 2*(b^2*c^2
*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*sqrt(c^2*x^2
+ 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^
2)*log(x))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2\,\text{arcsch}\,(cx)^2+2\,ab\,\text{arcsch}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{(a+b\,\text{acsch}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*acsch(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{(b\,\text{arcsch}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a)^2, x)
```

3.44 $\int (d+ex)^3 \left(a+b\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=167

$$\frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}\left(9c^2d^2-e^2\right)}{6c^3}+\frac{bd\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{2c^3}+\frac{bde^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c}+$$

```
[Out] (b*e*(9*c^2*d^2 - e^2)*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 +
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsch[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsch[c*x]))/(4*e) + (b*d*(2*
c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(2*c^3)
```

Rubi [A] time = 0.38363, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6290, 1568, 1475, 1807, 844, 215, 266, 63, 208}

$$\frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}\left(9c^2d^2-e^2\right)}{6c^3}+\frac{bd\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{2c^3}+\frac{bde^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```



```
[Out] (b*e*(9*c^2*d^2 - e^2)*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 +
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsch[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsch[c*x]))/(4*e) + (b*d*(2*
c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(2*c^3)
```


Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
```

```
2] ||  !IntegerQ[p])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{b\int \frac{(d+ex)^4}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{4ce}$$

$$= \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{b\int \frac{\left(e+\frac{d}{x}\right)^4x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{4ce}$$

$$= \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\,\text{Subst}\left(\int \frac{(e+dx)^4}{x^4\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4ce}$$

$$= \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{b\,\text{Subst}\left(\int \frac{-12de^3-2e^2\left(9d^2-\frac{e^2}{c^2}\right)x-12d^3ex}{x^3\sqrt{1+\frac{x^2}{c^2}}}\right.}{12ce}$$

$$= \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\,\text{Subst}\left(\int \frac{4e^2(}{}\right.}{}$$

$$= \frac{be\left(9c^2d^2-e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{(d+ex)^4\left(a+bc\right.}{4e}$$

$$= \frac{be\left(9c^2d^2-e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{(d+ex)^4\left(a+bc\right.}{4e}$$

$$= \frac{be\left(9c^2d^2-e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}-\frac{bd^4\text{csch}^{-1}(cx)}{4e}+$$

$$= \frac{be\left(9c^2d^2-e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}-\frac{bd^4\text{csch}^{-1}(cx)}{4e}+$$

$$= \frac{be\left(9c^2d^2-e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}-\frac{bd^4\text{csch}^{-1}(cx)}{4e}+$$

Mathematica [A] time = 0.279445, size = 165, normalized size = 0.99

$$\frac{3ac^3x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)+bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(18d^2+6dex+e^2x^2\right)-2e^2\right)+6bd\left(2c^2d^2-e^2\right)\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+\cdots\right.\right.}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 + 1/(c^
2*x^2)]*x*(-2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 +
6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsch[c*x] + 6*b*d*(2*c^2*d^2 - e^2)*L
og[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(12*c^3)
```

Maple [A] time = 0.178, size = 269, normalized size = 1.6

$$\frac{1}{c}\left(\frac{(cxe+cd)^4a}{4c^3e}+\frac{b}{c^3}\left(\frac{e^3\mathrm{arccsch}\,(cx)\,c^4x^4}{4}+e^2\mathrm{arccsch}\,(cx)\,c^4x^3d+\frac{3\,e\mathrm{arccsch}\,(cx)\,c^4x^2d^2}{2}+\mathrm{arccsch}\,(cx)\,c^4xd^3+\frac{\mathrm{arc}\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4*e^3*arccsch(c*x)*c^4*x^4+e^2*arcc
sch(c*x)*c^4*x^3*d+3/2*e*arccsch(c*x)*c^4*x^2*d^2+arccsch(c*x)*c^4*x*d^3+1/
4/e*arccsch(c*x)*c^4*d^4+1/12/e*(c^2*x^2+1)^(1/2)*(-3*c^4*d^4*arctanh(1/(c^
2*x^2+1)^(1/2))+12*c^3*d^3*e*arcsinh(c*x)+e^4*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c
^2*d*e^3*x*(c^2*x^2+1)^(1/2)+18*c^2*d^2*e^2*(c^2*x^2+1)^(1/2)-6*c*d*e^3*arc
sinh(c*x)-2*e^4*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.01557, size = 352, normalized size = 2.11

$$\frac{1}{4}ae^3x^4+ade^2x^3+\frac{3}{2}ad^2ex^2+\frac{3}{2}\left(x^2\,\mathrm{arcsch}\,(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2e+\frac{1}{4}\left(4\,x^3\,\mathrm{arcsch}\,(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1\cdots}\right)}{c^2}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arccsch(c*x) + x*s
qrt(1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*
x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^
2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arccsch(c*
x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e
^3 + a*d^3*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - l
og(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^3/c
```

Fricas [B] time = 3.64093, size = 898, normalized size = 5.38

$$\frac{3\,ac^3e^3x^4 + 12\,ac^3de^2x^3 + 18\,ac^3d^2ex^2 + 12\,ac^3d^3x + 3\left(4\,bc^3d^3 + 6\,bc^3d^2e + 4\,bc^3de^2 + bc^3e^3\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3*
d^3*x + 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*e^3)*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 6*(2*b*c^2*d^3 - b*d*e^2)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e +
4*b*c^3*d*e^2 + b*c^3*e^3)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1)
 + 3*(b*c^3*e^3*x^4 + 4*b*c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3*d^3*x
 - 4*b*c^3*d^3 - 6*b*c^3*d^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e^3*x^3 + 6*b*c^2*d*e^2*x^2 +
2*(9*b*c^2*d^2*e - b*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{acsch}\,(cx))\,(d + ex)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (b \operatorname{arcsch}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arccsch(c*x) + a), x)
```

3.45 $\int (d+ex)^2 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=122

$$\frac{(d+ex)^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e}+\frac{b\left(6c^2d^2-e^2\right)\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}+\frac{bdex\sqrt{\frac{1}{c^2x^2}+1}}{c}+\frac{be^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}-\frac{bd^3\,\text{csch}^{-1}(cx)}{3e}$$

```
[Out] (b*d*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsch[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsch[c*x]))/(3*e) +
(b*(6*c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rubi [A] time = 0.25939, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6290, 1568, 1475, 1807, 844, 215, 266, 63, 208}

$$\frac{(d+ex)^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e}+\frac{b\left(6c^2d^2-e^2\right)\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}+\frac{bdex\sqrt{\frac{1}{c^2x^2}+1}}{c}+\frac{be^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}-\frac{bd^3\,\text{csch}^{-1}(cx)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*d*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsch[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsch[c*x]))/(3*e) +
(b*(6*c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\int \frac{(d+ex)^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2} dx}{3ce}$$

$$= \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^3 x}{\sqrt{1+\frac{1}{c^2x^2}}} dx}{3ce}$$

$$= \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{b\,\text{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{3ce}$$

$$= \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\,\text{Subst}\left(\int \frac{-6de^2-e\left(6d^2-\frac{e^2}{c^2}\right)x-2d^3x^2}{x^2\sqrt{1+\frac{x^2}{c^2}}} dx\right.}{6ce}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{b\,\text{Subst}\left(\int \frac{e\left(6d^2-\right.}{x\sqrt{}}\right.}{6c}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{\left(bd^3\right)\text{Subst}\left(\int -\right.}{3ce}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} -$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} -$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} +$$

Mathematica [A] time = 0.181265, size = 122, normalized size = 1.

$$\frac{c^2x\left(2ac\left(3d^2+3dex+e^2x^2\right)+be\sqrt{\frac{1}{c^2x^2}+1}(6d+ex)\right)+b\left(6c^2d^2-e^2\right)\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)\right)+2bc^3x\text{csch}^{-1}(cx)\left(3d^2+\right.}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x*(b*e*Sqrt[1 + 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e^
2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsch[c*x] + b*(6*c^2*d^2
 - e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3)
```

Maple [A] time = 0.218, size = 204, normalized size = 1.7

$$\frac{1}{c}\left(\frac{(cxe+cd)^3 a}{3c^2e}+\frac{b}{c^2}\left(\frac{e^2\operatorname{arccsch}(cx)c^3x^3}{3}+e\operatorname{arccsch}(cx)c^3x^2d+\operatorname{arccsch}(cx)c^3xd^2+\frac{\operatorname{arccsch}(cx)c^3d^3}{3e}+\frac{1}{6cxe}\sqrt{c^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(1/3*e^2*arccsch(c*x)*c^3*x^3+e*arccsc
h(c*x)*c^3*x^2*d+arccsch(c*x)*c^3*x*d^2+1/3/e*arccsch(c*x)*c^3*d^3+1/6/e*(c
^2*x^2+1)^(1/2)*(-2*c^3*d^3*arctanh(1/(c^2*x^2+1)^(1/2))+6*c^2*d^2*e*arcsin
h(c*x)+e^3*c*x*(c^2*x^2+1)^(1/2)+6*c*d*e^2*(c^2*x^2+1)^(1/2)-e^3*arcsinh(c*
x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.02931, size = 259, normalized size = 2.12

$$\frac{1}{3}ae^2x^3+adex^2+\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bde+\frac{1}{12}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}\right.}{c^2}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*
b*d*e + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^
```

```
2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2)
+ 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c
^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^2/c
```

Fricas [B] time = 3.17135, size = 709, normalized size = 5.81

$$\frac{2\,ac^3e^2x^3 + 6\,ac^3dex^2 + 6\,ac^3d^2x + 2\left(3\,bc^3d^2 + 3\,bc^3de + bc^3e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - \left(6\,bc^2d^2 - be^2\right)\log\left(cx\sqrt{\frac{c^2x^2+}{c^2x^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(3*b*c^3*d^2 + 3
*b*c^3*d*e + b*c^3*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) -
(6*b*c^2*d^2 - b*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(3*b
*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) -
 c*x - 1) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^
2 - 3*b*c^3*d*e - b*c^3*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) + (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsch}(cx)) (d + ex)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (b \operatorname{arcsch}(cx) + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arccsch(c*x) + a), x)
```

3.46 $\int (d+ex)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=81

$$\frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}-\frac{bd^2\operatorname{csch}^{-1}(cx)}{2e}$$

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsch[c*x])/(2*e) + ((d + e*
x)^2*(a + b*ArcCsch[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rubi [A] time = 0.160567, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6290, 1568, 1396, 1807, 844, 215, 266, 63, 208}

$$\frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}-\frac{bd^2\operatorname{csch}^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsch[c*x])/(2*e) + ((d + e*
x)^2*(a + b*ArcCsch[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1396

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[((d + e/x^n)^q*(a + c/x^(2*n))^p)/x^2, x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)dx=\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{b\int\frac{(d+ex)^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{2ce}$$

$$=\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{b\int\frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{2ce}$$

$$=\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\,\text{Subst}\left(\int\frac{(e+dx)^2}{x^2\sqrt{1+\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{2ce}$$

$$=\frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{b\,\text{Subst}\left(\int\frac{-2de-d^2x}{x\sqrt{1+\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{2ce}$$

$$=\frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{(bd)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{c}-\frac{(bd^2}{}$$

$$=\frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\text{csch}^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{(bd)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{c^2}}}\right.}{2c}$$

$$=\frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\text{csch}^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-(bcd)\,\text{Subst}\left(\int\frac{}{-c^2}\right.$$

$$=\frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\text{csch}^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x}}\right.}{c}$$

Mathematica [A] time = 0.195236, size = 99, normalized size = 1.22

$$adx+\frac{1}{2}aex^2+\frac{bdx\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+\frac{bex\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2c}+bdx\text{csch}^{-1}(cx)+\frac{1}{2}bex^2\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*A
rcCsch[c*x] + (b*e*x^2*ArcCsch[c*x])/2 + (b*d*Sqrt[1 + 1/(c^2*x^2)]*x*ArcSi
nh[c*x])/Sqrt[1 + c^2*x^2]
```

Maple [A] time = 0.22, size = 115, normalized size = 1.4

$$\frac{1}{c}\left(\frac{a}{c}\left(\frac{c^2x^2e}{2}+c^2dx\right)+\frac{b}{c}\left(\frac{\operatorname{arccsch}(cx)\,c^2x^2e}{2}+\operatorname{arccsch}(cx)\,c^2xd+\frac{1}{2\,cx}\sqrt{c^2x^2+1}\left(e\sqrt{c^2x^2+1}+2\,cd\operatorname{Arcsinh}(cx)\right)\frac{1}{\sqrt{\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c*(a/c*(1/2*c^2*x^2*e+c^2*d*x)+b/c*(1/2*arccsch(c*x)*c^2*x^2*e+arccsch(c*
x)*c^2*x*d+1/2/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2+1)^(1/2)*(e*(c^2*x^
2+1)^(1/2)+2*c*d*arcsinh(c*x))))
```

Maxima [A] time = 1.00202, size = 117, normalized size = 1.44

$$\frac{1}{2}\,aex^2+\frac{1}{2}\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)be+adx+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b*e + a*d*
x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(
c^2*x^2) + 1) - 1))*b*d/c
```

Fricas [B] time = 2.65041, size = 479, normalized size = 5.91

$$\frac{acex^2+2\,acdx+bex\sqrt{\frac{c^2x^2+1}{c^2x^2}}-2\,bd\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)+(2\,bcd+bce)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-(2\,bcd+bce)\log\left(\ldots\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*e*x^2 + 2*a*c*d*x + b*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 2*b*d*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + (2*b*c*d + b*c*e)*log(c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (2*b*c*d + b*c*e)*log(c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*e*x^2 + 2*b*c*d*x - 2*b*c*d - b*c*e)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{acsch}\,(cx))\,(d + ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(b\,\mathrm{arcsch}\,(cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arccsch(c*x) + a), x)
```

3.47 $\int \left(a + b\operatorname{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=30

$$ax + \frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c} + bx\operatorname{csch}^{-1}(cx)$$

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rubi [A] time = 0.0220054, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6278, 266, 63, 208}

$$ax + \frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c} + bx\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 6278

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\,\mathrm{csch}^{-1}(cx)\right)dx &= ax+b\int \mathrm{csch}^{-1}(cx)\,dx\\
&= ax+bx\,\mathrm{csch}^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x}\,dx}{c}\\
&= ax+bx\,\mathrm{csch}^{-1}(cx)-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}\\
&= ax+bx\,\mathrm{csch}^{-1}(cx)-(bc)\,\mathrm{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)\\
&= ax+bx\,\mathrm{csch}^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [A] time = 0.0481137, size = 44, normalized size = 1.47

$$ax+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+bx\,\mathrm{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*Sqrt[1 + 1/(c^2*x^2)]*x*ArcSinh[c*x])/Sqrt[1 +
c^2*x^2]
```

Maple [A] time = 0.169, size = 36, normalized size = 1.2

$$ax+bx\,\mathrm{arccsch}\,(cx)+\frac{b}{c}\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsch(c*x),x)
```

```
[Out] a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

Maxima [A] time = 0.987604, size = 66, normalized size = 2.2

$$ax + \frac{\left(2\,cx\,\mathrm{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b/c
```

Fricas [B] time = 2.38492, size = 320, normalized size = 10.67

$$\frac{acx + bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (bcx - bc)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{acsch}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsch(c*x),x)
```

```
[Out] Integral(a + b*acsch(c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arcsch}(cx) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arccsch(c*x) + a, x)
```

3.48 $\int \frac{a+b\textbf{csch}^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=215

$$\frac{b\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{b\text{PolyLog}\left(2,\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{b\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2e}+\frac{\left(a+b\text{csch}^{-1}(c\right.}{}$$

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c
*d)])/e + ((a + b*ArcCsch[c*x])*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsc
h[c*x])/(c*d)])/e - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e +
(b*PolyLog[2, ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e + (b*Pol
yLog[2, ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e - (b*PolyLog[2
, E^(2*ArcCsch[c*x])])/(2*e)
```

Rubi [A] time = 0.389729, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6289, 2518}

$$\frac{b\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{b\text{PolyLog}\left(2,\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{b\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2e}+\frac{\left(a+b\text{csch}^{-1}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x),x]
```


```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c
*d)])/e + ((a + b*ArcCsch[c*x])*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsc
h[c*x])/(c*d)])/e - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e +
(b*PolyLog[2, ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e + (b*Pol
yLog[2, ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e - (b*PolyLog[2
, E^(2*ArcCsch[c*x])])/(2*e)
```


Rule 6289

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x
])/(c*d)])/e, x] + (Dist[b/(c*e), Int[Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^
ArcCsch[c*x])/(c*d)]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Dist[b/(c*e), In
```

```
t[Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)]/(x^2*Sqrt[1 + 1
/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2*ArcCsch[c*x])]/(x^2*S
qrt[1 + 1/(c^2*x^2)]), x], x] + Simp[((a + b*ArcCsch[c*x])*Log[1 - ((e + Sq
rt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e, x] - Simp[((a + b*ArcCsch[c*x
])*Log[1 - E^(2*ArcCsch[c*x])])/e, x]) /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(c}}{cd}\right.}{e}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(c}}{cd}\right.}{e}$$

Mathematica [C] time = 0.729286, size = 506, normalized size = 2.35

$$\frac{a\log(d+ex)}{e}+\frac{b\left(8\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+8\text{PolyLog}\left(2,\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+4\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 -
32*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*ArcTan[((I*c*d + e)*Cot[(Pi + (2*I
)*ArcCsch[c*x])/4])/Sqrt[c^2*d^2 + e^2]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*Arc
Csch[c*x])] + (4*I)*Pi*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/
(c*d)] + 8*ArcCsch[c*x]*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])
/(c*d)] + (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*Log[1 + ((-e + Sqrt[
c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + (4*I)*Pi*Log[1 - ((e + Sqrt[c^2*d^
2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*ArcCsch[c*x]*Log[1 - ((e + Sqrt[c^2*d^
```

```
2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt
[2]]*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (4*I)*Pi*L
og[e + d/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((e - Sqrt[c
^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*PolyLog[2, ((e + Sqrt[c^2*d^2 + e
^2])*E^ArcCsch[c*x])/(c*d)]))/(8*e)
```

Maple [F] time = 0.566, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{ex + d}\, dx + \frac{a \log(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x + d), x) + a*log(e*x
+ d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsch}(cx)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d), x)
```

3.49 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{d\sqrt{c^2d^2+e^2}}+\frac{b\,\mathrm{csch}^{-1}(cx)}{de}$$

```
[Out] (b*ArcCsch[c*x])/(d*e) - (a + b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c
^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2
 + e^2])
```

Rubi [A] time = 0.15371, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6290, 1568, 1475, 844, 215, 725, 206}

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{d\sqrt{c^2d^2+e^2}}+\frac{b\,\mathrm{csch}^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*ArcCsch[c*x])/(d*e) - (a + b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c
^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2
 + e^2])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
```

```
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)}\,dx}{ce}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}\,dx}{ce}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\,\mathrm{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cde}$$

$$= \frac{b\operatorname{csch}^{-1}(cx)}{de} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{d^2+\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d-\frac{e}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{cd}$$

$$= \frac{b\operatorname{csch}^{-1}(cx)}{de} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2+e^2}}$$

Mathematica [A] time = 0.214515, size = 134, normalized size = 1.37

$$-\frac{a}{e(d+ex)} - \frac{b\log\left(cx\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}-cd\right)+e\right)}{d\sqrt{c^2d^2+e^2}} + \frac{b\log(d+ex)}{d\sqrt{c^2d^2+e^2}} + \frac{b\sinh^{-1}\left(\frac{1}{cx}\right)}{de} - \frac{b\operatorname{csch}^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcSinh[1/(c*x)])/
(d*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 + e^2]) - (b*Log[e + c*(-(c*d) + S
qrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 + e^2])
```

Maple [B] time = 0.305, size = 208, normalized size = 2.1

$$-\frac{ac}{(cxe+cd)e}-\frac{b\mathrm{arccsch}(cx)}{(cxe+cd)e}+\frac{b}{cxed}\sqrt{c^2x^2+1}\mathrm{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}-\frac{b}{cxed}\sqrt{c^2x^2+1}\ln\left(2\frac{1}{cxe+cd}\left(\sqrt{\frac{c^2d^2}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arccsch(c*x)+1/c*b/e*(c^2*x^2+1)^(1/2)
/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d*arctanh(1/(c^2*x^2+1)^(1/2))-1/c*b/e*(c^2*
x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/((c^2*d^2+e^2)/e^2)^(1/2)*ln(2
*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-c^2*d*x+e)/(c*e*x+c*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(2c^2\int\frac{x}{c^2e^2x^3+c^2dex^2+e^2x+de+\left(c^2e^2x^3+c^2dex^2+e^2x+de\right)\sqrt{c^2x^2+1}}\,dx+\frac{ic(\log(icx+1)-\log(-icx+1))}{c^2d^2+e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*integrate(x/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e + (c^2*e^2
*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt(c^2*x^2 + 1)), x) + I*c*(log(I*c*x +
 1) - log(-I*c*x + 1))/(c^2*d^2 + e^2) - 2*e*log(e*x + d)/(c^2*d^3 + d*e^2)
 - (2*c^2*d^3*log(c) + 2*d*e^2*log(c) - 2*(c^2*d^2*e + e^3)*x*log(x) + (c^2
*d^2*e*x + c^2*d^3)*log(c^2*x^2 + 1) - 2*(c^2*d^3 + d*e^2)*log(sqrt(c^2*x^2
 + 1) + 1))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d*e^4)*x))*b - a/(e^2*x +
 d*e)
```

Fricas [B] time = 2.68307, size = 741, normalized size = 7.56

$$-\frac{ac^2d^3+ade^2-\sqrt{c^2d^2+e^2}\left(be^2x+bde\right)\log\left(-\frac{c^3d^2x-cde+\left(c^3d^2+ce^2\right)x\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\left(c^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+c^2dx-e\right)\sqrt{c^2d^2+e^2}}{ex+d}\right)-\left(bc^2d^3+bde^2+\left(\right.\right.}{c^2d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*e^2 - sqrt(c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*log(-(c^3*d^2
*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + (c^2*d*x*s
qrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 + e^2))/(e*x + d))
 - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*log(c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*
log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^2*d^3 + b*d*e^2)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e + d^2*e^3 + (c
^2*d^3*e^2 + d*e^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acsch}(cx)}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcsch}(cx) + a}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^2, x)
```

3.50 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}-\frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{2d\left(c^2d^2+e^2\right)\left(\frac{d}{x}+e\right)}+\frac{b\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}+\frac{b\text{csch}^{-1}(cx)}{2d^2e}$$

```
[Out] -(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(2*d*(c^2*d^2 + e^2)*(e + d/x)) + (b*ArcCsch
[c*x])/(2*d^2*e) - (a + b*ArcCsch[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2 +
 e^2)*ArcTanh[(c^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])
/(2*d^2*(c^2*d^2 + e^2)^(3/2))
```

Rubi [A] time = 0.29184, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6290, 1568, 1475, 1651, 844, 215, 725, 206}

$$-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}-\frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{2d\left(c^2d^2+e^2\right)\left(\frac{d}{x}+e\right)}+\frac{b\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}+\frac{b\text{csch}^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

```
[Out] -(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(2*d*(c^2*d^2 + e^2)*(e + d/x)) + (b*ArcCsch
[c*x])/(2*d^2*e) - (a + b*ArcCsch[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2 +
 e^2)*ArcTanh[(c^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])
/(2*d^2*(c^2*d^2 + e^2)^(3/2))
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^2}\,dx}{2ce}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2x^4}\,dx}{2ce}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{(e+dx)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{(bc)\,\mathrm{Subst}\left(\int \frac{e-\left(d+\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2e} - \frac{\left(bc\left(2+\frac{e^2}{c^2d^2}\right)\right)\mathrm{Sub}}{2(}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(bc\left(2+\frac{e^2}{c^2d^2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{d^2+\frac{e^2}{c^2}-x^2}\right.}{2\left(c^2d^2+e^2\right)}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1}}\right.}{2d^2\left(c^2d^2+e^2\right)^{3/2}}$$

Mathematica [A] time = 0.46198, size = 204, normalized size = 1.25

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bcex\sqrt{\frac{1}{c^2x^2}+1}}{d\left(c^2d^2+e^2\right)(d+ex)} - \frac{b\left(2c^2d^2+e^2\right)\log\left(cx\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}-cd\right)+e\right)}{d^2\left(c^2d^2+e^2\right)^{3/2}} + \frac{b\left(2c^2d^2+e^2\right)\log(d-}{d^2\left(c^2d^2+e^2\right)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d^2 + e^2)*
(d + e*x)) - (b*ArcCsch[c*x])/(e*(d + e*x)^2) + (b*ArcSinh[1/(c*x)])/(d^2*e
) + (b*(2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c^2*d^2 + e^2)^(3/2)) - (b*(2*
c^2*d^2 + e^2)*Log[e + c*(-(c*d) + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)
])*x])/(d^2*(c^2*d^2 + e^2)^(3/2)))/2
```

Maple [B] time = 0.273, size = 963, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arccsch(c*x)+1/2*c^2*b
*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ar
ctanh(1/(c^2*x^2+1)^(1/2))+1/2*c^2*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/x*d/(c^2*d^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))-c^2*b*
(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/((c^2*d^2+e^2)/e^2)^(1/2)/(c^
2*d^2+e^2)/(c*e*x+c*d)*ln(2*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-
c^2*d*x+e)/(c*e*x+c*d))-c^2*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/
2)/x*d/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ln(2*(((c^2*d^2+
e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-c^2*d*x+e)/(c*e*x+c*d))-1/2*c^2*b*e/((c
^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*d^2+e^2)/(c*e*x+c*d)-1/2*b*e/((c^2*x^2+1)
/c^2/x^2)^(1/2)/x/d/(c^2*d^2+e^2)/(c*e*x+c*d)+1/2*b*e^2*(c^2*x^2+1)^(1/2)/(
(c^2*x^2+1)/c^2/x^2)^(1/2)/d^2/(c^2*d^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2
+1)^(1/2))+1/2*b*e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d
^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))-1/2*b*e^2*(c^2*x^2+1)^(1/2
)/((c^2*x^2+1)/c^2/x^2)^(1/2)/d^2/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(
c*e*x+c*d)*ln(2*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-c^2*d*x+e)/(
c*e*x+c*d))-1/2*b*e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/((c^2
*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ln(2*(((c^2*d^2+e^2)/e^2)^(1
/2)*(c^2*x^2+1)^(1/2)*e-c^2*d*x+e)/(c*e*x+c*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\left(\frac{2i\,c^3d(\log\left(i\,cx+1\right)-\log\left(-i\,cx+1\right))}{c^4d^4+2\,c^2d^2e^2+e^4}+4\,c^2\int\frac{x}{2\left(c^2e^3x^4+2\,c^2de^2x^3+2\,de^2x+d^2e+\left(c^2d^2e+e^3\right)x^2+\left(c^2e^3x^4+2\,c^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*I*c^3*d*(log(I*c*x + 1) - log(-I*c*x + 1))/(c^4*d^4 + 2*c^2*d^2*e^2
 + e^4) + 4*c^2*integrate(1/2*x/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x
+ d^2*e + (c^2*d^2*e + e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*
x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(3*c^2*d^2*e
+ e^3)*log(e*x + d)/(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4) - (2*c^4*d^6*log(c)
 + 2*d^2*e^4*log(c) - 2*d^2*e^4 + 2*(2*d^4*e^2*log(c) - d^4*e^2)*c^2 - 2*(c
^2*d^3*e^3 + d*e^5)*x + (c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4
)*x^2 + 2*(c^4*d^5*e - c^2*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*((c^4*d^4*e^2 +
 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*x)*log(x)
 - 2*(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d
^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^
2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x))*b - 1/2*a/(e^3*x^2 + 2*d*
e^2*x + d^2*e)
```

Fricas [B] time = 5.63123, size = 1524, normalized size = 9.35

$$-\frac{ac^4d^6+bc^3d^5e+2\,ac^2d^4e^2+bcd^3e^3+ad^2e^4+\left(bc^3d^3e^3+bcde^5\right)x^2-\left(2\,bc^2d^4e+bd^2e^3+\left(2\,bc^2d^2e^3+be^5\right)x^2+2\left(2\,bc^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^4*d^6 + b*c^3*d^5*e + 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4 +
 (b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e + b*d^2*e^3 + (2*b*c^2*d^
2*e^3 + b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 + b*d*e^4)*x)*sqrt(c^2*d^2 + e^2)*l
og(-(c^3*d^2*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ (c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 + e^2)
)/(e*x + d)) + 2*(b*c^3*d^4*e^2 + b*c*d^2*e^4)*x - (b*c^4*d^6 + 2*b*c^2*d^4
```

```
*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4
*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) - c*x + 1) + (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 +
2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*
x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^4*d^6 + 2*b*c^2*
d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (
(b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 + b*c*d^2*e^4)*x)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3
+ 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsch}(cx)}{(d + ex)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^3, x)
```

3.51 $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=918

$$-\frac{32b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}+\frac{32bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\right.}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] (-4*b*d*Sqrt[d + e*x]*(1 + c^2*x^2))/(105*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) +
(4*b*(d + e*x)^(3/2)*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(
a + b*ArcCsch[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*
e^3) - (32*b*c*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(
-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[
-c^2]*e)]) - (4*b*c*(c^2*d^2 - 3*e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqr
t[-c^2]*e)])/(35*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*
x))/(c^2*d - Sqrt[-c^2]*e)]) + (32*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sq
rt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt
[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(3/2)*e^2*Sqrt
[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-
c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(Sqr
t[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(105*c*e^3*S
qrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 3.2528, antiderivative size = 918, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {43, 6310, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537, 833}

$$-\frac{32b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}+\frac{32bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c}}{c^2d-\sqrt{-}}\right.}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (-4*b*d*Sqrt[d + e*x]*(1 + c^2*x^2))/(105*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) +
(4*b*(d + e*x)^(3/2)*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(
a + b*ArcCsch[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*
e^3) - (32*b*c*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(
-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[
-c^2]*e)]) - (4*b*c*(c^2*d^2 - 3*e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqr
t[-c^2]*e)])/(35*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*
x))/(c^2*d - Sqrt[-c^2]*e)]) + (32*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sq
rt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt
[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(3/2)*e^2*Sqrt
[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-
c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(Sqr
t[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(105*c*e^3*S
qrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)dx = \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/}}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/}}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/}}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/}}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/}}{}$$

$$= -\frac{16bd\sqrt{d+ex}\left(1+c^2x^2\right)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-}\right.}{3e^3}$$

$$= -\frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}\right.}{3e^3}$$

$$= -\frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}\right.}{3e^3}$$

$$= -\frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}\right.}{3e^3}$$

$$= -\frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}\right.}{3e^3}$$

$$\frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{} \quad \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{} \quad \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}\right.}{}$$

Mathematica [C] time = 14.284, size = 1094, normalized size = 1.19

$$b\left(-\frac{c\left(\frac{d}{x}+e\right)x\left(-\frac{16c^3\operatorname{csch}^{-1}(cx)d^3}{105e^3}-\frac{2}{7}c^3x^3\operatorname{csch}^{-1}(cx)-\frac{2c^2x^2\left(2\sqrt{1+\frac{1}{c^2x^2}}e+cd\operatorname{csch}^{-1}(cx)\right)}{35e}-\frac{8cx\left(cde\sqrt{1+\frac{1}{c^2x^2}}-c^2d^2\operatorname{csch}^{-1}(cx)\right)}{105e^2}+\frac{4\left(5c^2d^2+9e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{105e^2}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{\frac{d}{x}+e}\sqrt{cx}\left(-\ldots\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d])) +
 (b*(-((c*(e + d/x)*x*((4*(5*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*e
^2) - (16*c^3*d^3*ArcCsch[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsch[c*x])/7 - (2
*c^2*x^2*(2*e*Sqrt[1 + 1/(c^2*x^2)] + c*d*ArcCsch[c*x]))/(35*e) - (8*c*x*(c
*d*e*Sqrt[1 + 1/(c^2*x^2)] - c^2*d^2*ArcCsch[c*x]))/(105*e^2)))/Sqrt[d + e*
x]) - (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(9*c^3*d^3*e + c*d*e^3)*Sqrt[1
 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*S
qrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(
c*d - I*e)*(8*c^4*d^4 - 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I +
 c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-5*c^3*d^3*e - 9*c*d*e^3)*Cosh[2*ArcCsch[
c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I +
c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/
(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*
(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d +
e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)
*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^
```

```
2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

Maple [C] time = 0.684, size = 2515, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(1
/7*arccsch(c*x)*(e*x+d)^(7/2)-2/5*arccsch(c*x)*d*(e*x+d)^(5/2)+1/3*arccsch(
c*x)*d^2*(e*x+d)^(3/2)+2/105/c^4*(-I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x
+d)^(1/2)*c*d*e^3-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(7/2)*c^4*d-8
*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*
x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)
^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I
*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e
+7*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^4*d^2+5*I*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^3*d^2*e-4*(-(I*(e*x+d)*c*e+(e*x+d)*c^
2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2
+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-5*(-(I*(e
*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(
e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
))*c^4*d^4+9*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(
1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e+8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),
1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)
*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4+I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e
^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^
2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(
2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3-5*((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2)*(e*x+d)^(3/2)*c^4*d^3-I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*
x+d)^(1/2)*c^3*d^3*e+((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^4*d^
4-7*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^3*d*e+13*(-(I*(e*x+
d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x
+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c
```

```
*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*
c^2*d^2*e^2-14*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(
1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipt
icE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2+3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*
(e*x+d)^(7/2)*c^3*e-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d
*e^2+3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c*e^3+((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2*e^2+9*(-(I*(e*x+d)*c*e+(e*x+d)*
c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4-9*(-(I*(e*x
+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*
x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*e^4)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}(b\,\mathrm{arcsch}\,(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x^2, x)
```

3.52 $\int x\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=679

$$\frac{4bc\sqrt{c^2x^2+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{8bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}\left(\mathrm{si}\right.}{15\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+\ldots}}$$

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d*(
d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
cCsch[c*x]))/(5*e^2) + (8*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[A
rcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2
]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2
*d - Sqrt[-c^2]*e)]) - (8*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*
e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2
*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*
x^2)]*x*Sqrt[d + e*x]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]
/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x])
```

Rubi [A] time = 2.51914, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.79, Rules used = {43, 6310, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537}

$$-\frac{2d(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5e^2}+\frac{4bc\sqrt{c^2x^2+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\ldots}}{\sqrt{\ldots}}\right.\right.}{15\left(-c^2\right)^{5/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d*(
d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
```

```
cCsch[c*x]))/(5*e^2) + (8*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[A
rcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2
]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2
*d - Sqrt[-c^2]*e)]) - (8*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*
e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2
*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*
x^2)]*x*Sqrt[d + e*x]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]
/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)dx = -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{b\int\frac{2(d+ex)^{3/2}(-2}{15e^2\sqrt{1+\frac{}{c}}}}{c}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{(2b)\int\frac{(d+ex)^{3/2}}{\sqrt{1+}}}{15ce}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(2b\sqrt{1+c^2x^2}\right)}{15ce}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(2b\sqrt{1+c^2x^2}\right)}{1}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}-\frac{\left(4bd\sqrt{1+c^2x^2}\right)}{15ce^2\sqrt{}}$$

$$=\frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}\right)}{5e^2}$$

$$=\frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}\right)}{5e^2}$$

$$=\frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}\right)}{5e^2}$$

$$=\frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}\right)}{5e^2}$$

$$=\frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}\right)}{5e^2}$$

$$=\frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}\right)}{5e^2}$$

Mathematica [C] time = 1.60524, size = 418, normalized size = 0.62

$$\frac{1}{15}\left(\frac{4ib\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}\left(\left(c^2d^2-2icde+e^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right),\frac{cd-ie}{cd+ie}\right)-2c^2d^2\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\right.\right.}{c^3e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-\frac{c}{cd-ie}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] ((4*b*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2
 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2
)*ArcCsch[c*x])/e^2 + ((4*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((
e*(I + c*x))/(c*d - I*e))]*(2*c*d*(c*d + I*e)*EllipticE[I*ArcSinh[Sqrt[-(c/
(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (c^2*d^2 - (2*I)*c
*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d
 - I*e)/(c*d + I*e)] - 2*c^2*d^2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt
[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^3*Sqrt[-(c
/(c*d - I*e))]*e^2*Sqrt[1 + 1/(c^2*x^2)]*x))/15
```

Maple [C] time = 0.32, size = 1964, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e^2*(a*(1/5*(e*x+d)^(5/2)-1/3*d*(e*x+d)^(3/2))+b*(1/5*arccsch(c*x)*(e*x+d
)^(5/2)-1/3*arccsch(c*x)*d*(e*x+d)^(3/2)+2/15/c^3*(2*I*(-(I*(e*x+d)*c*e+(e*
x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2)
)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2+
e^2))^(1/2)*(e*x+d)^(5/2)*c^3*d-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-
(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+2*((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^3*d^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*
(e*x+d)^(1/2)*e^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2
*e+(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*
```

```
x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^
(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2))*c^3*d^3+2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c
^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3-I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3-((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2)*(e*x+d)^(1/2)*c^3*d^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*
x+d)^(5/2)*c^2*e-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))
^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Elli
pticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2+2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(
-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-2*I*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(
e*x+d)^(1/2)*c*d*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/
e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x, x)
```

3.53 $\int \sqrt{d+ex}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=429

$$\frac{4bcd\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{2(d+ex)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{c^2x^2+1}\sqrt{\sqrt{\ldots}}}{3\ldots}$$

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (4*b*c*Sqrt[d + e*x]*Sqrt[
1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^
2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*c*d*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2]
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e
)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
(2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 0.708368, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6290, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bcd\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}F\ldots}{3\left(-c^2\right)^{3/2}x\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (4*b*c*Sqrt[d + e*x]*Sqrt[
1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^
2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*c*d*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2]
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e
)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
```

```
(2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
```

```
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)dx = \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{(2b)\int \frac{(d+ex)^{3/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{3ce}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int\left(\frac{2de}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \cdots\right.}{3ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(4bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2bd^2\sqrt{\frac{1}{c^2}+x\right.\cdots}{3ce\cdots}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \cdots}{3c\sqrt{1+\frac{1}{c^2\cdots}}\cdots}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{8b\sqrt{-c^2}d\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle| \cdots\right.}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle| -\cdots\right.}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle| -\cdots\right.}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$

Mathematica [C] time = 13.9916, size = 926, normalized size = 2.16

$$\frac{2a(d+ex)^{3/2}}{3e} + \frac{b\left(-\frac{(cd+cex)\left(-\frac{2}{3}cx\operatorname{csch}^{-1}(cx)-\frac{2cd\operatorname{csch}^{-1}(cx)}{3e}-\frac{4}{3}\sqrt{1+\frac{1}{c^2x^2}}\right)}{\sqrt{d+ex}} - \frac{2(cd+cex)\left(-\frac{\sqrt{2}cde\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\operatorname{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right),\frac{icd+e}{2e}\right)}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}+\frac{i\sqrt{2}\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) + (b*(-(((c*d + c*e*x)*((-4*Sqrt[1 + 1/(c^2*x^2
)])/3 - (2*c*d*ArcCsch[c*x])/(3*e) - (2*c*x*ArcCsch[c*x])/3))/Sqrt[d + e*x]
) - (2*(c*d + c*e*x)*(-((Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d
+ c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
 (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[
(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(c^2*d^2 + e^2)*Sqr
t[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
 (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])
/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cosh[2*ArcCsch[
c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I +
c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/
(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*
(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d +
e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)
*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2)))
)/(3*e*Sqrt[e + d/x]*Sqrt[c*x]*Sqrt[d + e*x])))/c^2
```

Maple [C] time = 0.301, size = 840, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(1/3*a*(e*x+d)^(3/2)+b*(1/3*(e*x+d)^(3/2)*arccsch(c*x)+2/3/c^2*(-(I*(e*
x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e
*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(I*EllipticF((e*x+d)^(1/2)*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2))*c*d*e-2*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2
*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticE((e*x+d)^(1/2)
*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2))*c^2*d^2-I*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2))*c*d*e+EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))
^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticF((e*x+d)^(1/2)*(
(I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2))*e^2+EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*
c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2
*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e
-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a), x)
```

3.54 $\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi [A] time = 0.0760624, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A] time = 18.5683, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [A] time = 6.829, size = 0, normalized size = 0.

$$\int \frac{a + b\text{arccsch}\,(cx)}{x}\sqrt{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(b\,\text{arcsch}\,(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

3.55 $\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi [A] time = 0.0789321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 7.77065, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [A] time = 4.661, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^2}\sqrt{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex + d}(b\,\mathrm{arcsch}\,(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(b \operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.56 $\int (d+ex)^{3/2} \left(a+b\,\textbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=486

$$-\frac{4bc\sqrt{c^2x^2+1}\left(2c^2d^2-e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{2(d+ex)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{4bd^3}{\quad}$$

```
[Out] (4*b*e*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*(
d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (28*b*c*d*Sqrt[d + e*x]*Sqrt[1
 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2
]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) - (4*b*c*(2*c^2*d^2 - e^2)*Sqrt[(d + e*x)/(
d + e/Sqrt[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*Sqr
t[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/
(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(5*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[d + e*x])
```

Rubi [A] time = 1.02263, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6290, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d+ex)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{4bc\sqrt{c^2x^2+1}\left(2c^2d^2-e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4bd^3\sqrt{c^2x^2+1}}{\quad}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```


```
[Out] (4*b*e*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*(
d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (28*b*c*d*Sqrt[d + e*x]*Sqrt[1
 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2
]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) - (4*b*c*(2*c^2*d^2 - e^2)*Sqrt[(d + e*x)/(
d + e/Sqrt[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
```

```
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*Sqr
t[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/
(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(5*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[d + e*x])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)dx = \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{(2b)\int \frac{(d+ex)^{5/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{5ce}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int\left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \right.}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(6bd^2\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2bd^3\sqrt{\frac{1}{c^2}\right.}{5}$$

$$= \frac{4be\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(6bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+}}{\sqrt{\frac{1}{c^2}+}}}{5c\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4be\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{12b\sqrt{-c^2}d^2\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{}}{5c}$$

$$= \frac{4be\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1-}}{5c^3}$$

$$= \frac{4be\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1-}}{5c^3}$$

$$28b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1-}$$

Mathematica [C] time = 1.52612, size = 380, normalized size = 0.78

$$\frac{2\left(\frac{2ib\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}\left(\left(-9c^2d^2-7icde+e^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right),\frac{cd-ie}{cd+ie}\right)+3c^2d^2\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+7cd(cd+ie)E\left(i\sinh^{-1}\right.\right.}{c^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-\frac{c}{cd-ie}}}\right.}{15e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*((2*b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + 3*a*(d + e*x)^(5/2)
 + 3*b*(d + e*x)^(5/2)*ArcCsch[c*x] + ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d +
 I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(7*c*d*(c*d + I*e)*EllipticE[I*A
rcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (-
9*c^2*d^2 - (7*I)*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*S
qrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + 3*c^2*d^2*EllipticPi[1 - (I*e)/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*
e)]))/(c^3*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(15*e)
```

Maple [C] time = 0.293, size = 1939, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] 2/e*(1/5*(e*x+d)^(5/2)*a+b*(1/5*arccsch(c*x)*(e*x+d)^(5/2)+2/15/c^3*(I*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2)*(e*x+d)^(5/2)*c^3*d+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d
)^(5/2)*c^2*e+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^3*d^2-2*I
*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-9*(-(I*(e*x+d)*c*e
+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^
2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^
3+7*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e
*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)
^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2))*c^3*d^3+2*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
```

```
1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e
-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+3*(-(I*(e*x+d)*c*e+(e*x+d)*c^
2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2
+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)
/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3-I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*
d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3-((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^3*d^3-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d
-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-6*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d
-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2+7*(-(I*(e*x+
d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x
+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*
c*d*e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*e^3-((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c*d*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d
+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c
*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((aex + ad + (bex + bd)\,\text{arcsch}\,(cx))\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x + a*d + (b*e*x + b*d)*arccsch(c*x))*sqrt(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{3}{2}}(b\,\mathrm{arcsch}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

3.57 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=939

$$\frac{64b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)d^3}{e^4}-\frac{64bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2}}{35}$$

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]) - (4*b*d
*Sqrt[d + e*x]*(1 + c^2*x^2))/(21*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d^3
*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) + (2*
(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^4) + (24*b*c*d^2*Sqrt[d + e*x]*S
qrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt
[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2
)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*(2*c^2*d^2 + 9*
e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^
3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
64*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d -
 Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (32*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x
^2)]*x*Sqrt[d + e*x]) + (64*b*d^4*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
 + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2
]], (2*e)/(Sqrt[-c^2]*d + e)])/(35*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rubi [A] time = 2.87133, antiderivative size = 939, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.81, Rules used = {43, 6310, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844, 942, 1654}

$$\frac{64b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)d^3}{e^4}-\frac{64bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2}}{35\left(-c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]) - (4*b*d
*Sqrt[d + e*x]*(1 + c^2*x^2))/(21*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d^3
*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) + (2*
(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^4) + (24*b*c*d^2*Sqrt[d + e*x]*S
qrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt
[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2
)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*(2*c^2*d^2 + 9*
e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^
3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
64*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d -
 Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (32*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x
^2)]*x*Sqrt[d + e*x]) + (64*b*d^4*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
 + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2
]], (2*e)/(Sqrt[-c^2]*d + e)])/(35*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
```

```
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 942

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x])/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d+ex)^{5/2}\left(a+bc\right.}{5e^4}$$

$$= -\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d+ex)^{5/2}\left(a+bc\right.}{5e^4}$$

$$= -\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d+ex)^{5/2}\left(a+bc\right.}{5e^4}$$

$$= -\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d+ex)^{5/2}\left(a+bc\right.}{5e^4}$$

$$= -\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d+ex)^{5/2}\left(a+bc\right.}{5e^4}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{8bd\sqrt{d+ex}\left(1+c^2x^2\right)}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}\left(1+c^2x^2\right)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+}{}$$

Mathematica [C] time = 14.1763, size = 1098, normalized size = 1.17

$$\frac{a\sqrt{\frac{ex}{d}+1}B_{-\frac{ex}{d}}\left(4,\frac{1}{2}\right)d^4}{e^4\sqrt{d+ex}}+\frac{b\left(\frac{2\sqrt{\frac{d}{x}+e}\sqrt{cx}\left(-\frac{\sqrt{2}(40c^3d^3e-8cde^3)\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right),\frac{icd+e}{2e}\right)}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}+\frac{i\sqrt{2}(cd-ie)(48c^4d^4-16c^2e^2d^2+9e^4)\sqrt{icx+\cdots}}{e\sqrt{1\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x)*x*((4*(-16*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsch[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsch[c*x])/(7*e) -
(4*c^2*x^2*(e*Sqrt[1 + 1/(c^2*x^2)] - 3*c*d*ArcCsch[c*x]))/(35*e^2) + (4*c*
x*(5*c*d*e*Sqrt[1 + 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsch[c*x]))/(105*e^3)))/Sq
rt[d + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(40*c^3*d^3*e - 8*c*d
*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[A
rcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/
(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) +
(I*Sqrt[2]*(c*d - I*e)*(48*c^4*d^4 - 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[1 + I*c*x
]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e,
 ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1
+ 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-16*c^3*d^3*e + 9*c*d*e^3)*
Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (
2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))
/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellip
ticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*
e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*
(I + c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*x])))/c^4
```

Maple [C] time = 0.321, size = 2543, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e^4*(a*(1/7*(e*x+d)^(7/2)-3/5*d*(e*x+d)^(5/2)+d^2*(e*x+d)^(3/2)-d^3*(e*x+
d)^(1/2))+b*(1/7*arccsch(c*x)*(e*x+d)^(7/2)-3/5*arccsch(c*x)*d*(e*x+d)^(5/2
)+arccsch(c*x)*d^2*(e*x+d)^(3/2)-arccsch(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4*(
3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(7/2)*c^3*e-3*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2)*(e*x+d)^(7/2)*c^4*d+19*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2)*(e*x+d)^(3/2)*c^3*d^2*e+14*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/
2)*c^4*d^2-14*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^3*d*e-19*
((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^4*d^3-8*I*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c*d*e^3+24*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4+16*(-(I*(e*x+
d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x
+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*
c^4*d^4+3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c*e^3-48*(-(I*(
e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-
(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c
/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4+48*I*(-(I*
(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e
-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*(
(I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*
c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e+8*((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^4*d^4-40*I*(-(I*(e*x+d)*c*e+(e
*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d
+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e
-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e^2-8*I*((I*e+c*d)
*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^3*d^3*e-15*(-(I*(e*x+d)*c*e+(e*x+d)
*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2+7*
(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d
)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/
```

```
2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2))*c^2*d^2*e^2+8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e
-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3+8*((I*e+c*d)*c/(c^2*d^2+e^2))^(
1/2)*(e*x+d)^(1/2)*c^2*d^2*e^2+9*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4-9*(-(I*(e*x+d)*c*e+(e*x+d)*c
^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4)/(((e*x+d)^2
*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3\,\text{arcsch}\,(cx) + ax^3}{\sqrt{ex+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/sqrt(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^3}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x + d), x)
```

3.58 $\int \frac{x^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=707

$$\frac{32bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}}{15\left(-c^2\right)^{5/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*d
^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^3) - (
4*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2
]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(5*(-c^2)^(3/2)*e
^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) +
(32*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*
EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d
- Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x
]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sq
rt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[
-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (32*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
)
```

Rubi [A] time = 2.14168, antiderivative size = 707, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {43, 6310, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844}

$$\frac{2d^2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{32bcd^2\sqrt{c^2x^2+1}}{15(}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*d
^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^3) - (
4*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2
]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(5*(-c^2)^(3/2)*e
^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) +
(32*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*
EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d
- Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x
]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sq
rt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[
-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (32*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
```

```
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex}}\, dx = \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a + b\text{csc}\right.}{5e^3}$$

$$= \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a + b\text{csc}\right.}{5e^3}$$

$$= \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a + b\text{csc}\right.}{5e^3}$$

$$= \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a + b\text{csc}\right.}{5e^3}$$

$$= \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a + b\text{csc}\right.}{5e^3}$$

$$= \frac{4b\sqrt{d+ex}\left(1 + c^2x^2\right)}{15c^3e\sqrt{1 + \frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= \frac{4b\sqrt{d+ex}\left(1 + c^2x^2\right)}{15c^3e\sqrt{1 + \frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= \frac{4b\sqrt{d+ex}\left(1 + c^2x^2\right)}{15c^3e\sqrt{1 + \frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= \frac{4b\sqrt{d+ex}\left(1 + c^2x^2\right)}{15c^3e\sqrt{1 + \frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= \frac{4b\sqrt{d+ex}\left(1 + c^2x^2\right)}{15c^3e\sqrt{1 + \frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a + b\text{csch}^{-1}(cx)\right)}{3e^3}$$

Mathematica [C] time = 14.325, size = 1012, normalized size = 1.43

$$b\left(-\frac{c\left(\frac{d}{x}+e\right)x\left(-\frac{16c^2\operatorname{csch}^{-1}(cx)d^2}{15e^3}+\frac{4c\sqrt{1+\frac{1}{c^2x^2}}d}{5e^2}-\frac{2c^2x^2\operatorname{csch}^{-1}(cx)}{5e}-\frac{4cx\left(e\sqrt{1+\frac{1}{c^2x^2}}-2cd\operatorname{csch}^{-1}(cx)\right)}{15e^2}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{\frac{d}{x}+e}\sqrt{cx}\left(-\frac{\sqrt{2}(7c^2d^2e-e^3)\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\operatorname{EllipticF}\left(\sin^{-1}\left(\ldots\right.\right.}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}\ldots\right.}{\ldots}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] -((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x])) +
 (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^2*
ArcCsch[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsch[c*x])/(5*e) - (4*c*x*(e*Sqrt[1
+ 1/(c^2*x^2)] - 2*c*d*ArcCsch[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[e
 + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(7*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)
*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d
 - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^
(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(8*c^3*d
^3 - 3*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (
6*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sq
rt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSi
n[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I
 + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*
e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*
e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c
*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I
*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d
)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sq
rt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x])))/c^3
```

Maple [C] time = 0.305, size = 1991, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*d*(e*x+d)^(3/2)+d^2*(e*x+d)^(1/2))+b*(1/5*a
rccsch(c*x)*(e*x+d)^(5/2)-2/3*arccsch(c*x)*d*(e*x+d)^(3/2)+arccsch(c*x)*d^2
*(e*x+d)^(1/2)+2/15/c^3*(7*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2)*(e*x+d)^(5/2)*c^3*d+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/
2)*c^2*d^2*e+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*e^3-I*(-(I*(
e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-
(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2))*e^3+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^3*d^2-4*(-(I*(e
*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(
e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
))*c^3*d^3-3*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/
2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Elliptic
E((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2))*c^3*d^3+8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I
*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(
c^2*d^2+e^2))^(1/2))*c^3*d^3-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^
(3/2)*c^2*d*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^3*d^3+I*((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^2*e+4*(-(I*(e*x+d)*c*e+(e*x+
d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-3*(-
(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*
c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)
*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2))*c*d*e^2-8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^
2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*E
llipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^
2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2))*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c*d*e^2)/
```

```
(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x + d), x)
```

3.59 $\int \frac{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=474

$$-\frac{8bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{2d\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+\right.}{3e^2}$$

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*A
rcCsch[c*x]))/(3*e^2) + (4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[Ar
cSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]
*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]) - (8*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)
]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (
2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 1.75063, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {43, 6310, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537}

$$-\frac{2d\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^2}+\frac{8bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\right.}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*A
rcCsch[c*x]))/(3*e^2) + (4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[Ar
cSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]
*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]) - (8*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
```

```
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)
]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (
2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{b\int\frac{2(-2d+ex)\sqrt{d+ex}}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{(2b)\int\frac{(-2d+ex)\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{3ce^2}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{(-2d+ex)}{x\sqrt{1+}}}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(\frac{e\sqrt{d+e}}{\sqrt{1+c^2}}\right.}{3ce^2\sqrt{1+}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{\left(4bd\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+e}}{x\sqrt{1+c^2}}}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\int\frac{}{x\sqrt{d+}}}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2}}{3c^3e\sqrt{1}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2}}{3c^3e\sqrt{1}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2}}{3c^3e\sqrt{1}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2}}{3c^3e\sqrt{1}}$$

Mathematica [C] time = 1.27859, size = 343, normalized size = 0.72

$$\frac{2\left(\frac{2b\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}\left((e+icd)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right),\frac{cd-ie}{cd+ie}\right)+(-e+icd)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)-2icd\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+\right.\right.\right.}{c^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-\frac{c}{cd-ie}}}}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*(-2*d + e*x)*Sqrt[d + e*x] + b*(-2*d + e*x)*Sqrt[d + e*x]*ArcCsch[c*x
] + (2*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*
e))]*((I*c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]]
, (c*d - I*e)/(c*d + I*e)] + (I*c*d + e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d
- I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - (2*I)*c*d*EllipticPi[1
- (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)
/(c*d + I*e)]))/(c^2*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(3*e
^2)
```

Maple [C] time = 0.299, size = 868, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e^2*(a*(1/3*(e*x+d)^(3/2)-d*(e*x+d)^(1/2))+b*(1/3*(e*x+d)^(3/2)*arccsch(c
*x)-arccsch(c*x)*d*(e*x+d)^(1/2)-2/3/c^2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2)*(2*I*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-EllipticF((e*
x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-2*I*Ellipt
icPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2
+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
))*c*d*e+2*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*
e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE((e
```

```
*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2))*e^2)/((((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^
2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{acsch}\left(cx\right)\right)}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x + d), x)
```

3.60 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=284

$$\frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e}-\frac{4bd\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+}{\sqrt{-c^2}d}}}{cex\sqrt{}}$$

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e + (4*b*c*Sqrt[(d + e*x)/(d + e/Sqr
t[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*Sqrt[1 + 1/(c^2
*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
+ e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 0.413714, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6290, 1574, 944, 719, 419, 933, 168, 538, 537}

$$\frac{2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e}+\frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4bd\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2\right.}{cex\sqrt{\frac{1}{c^2x^2}+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e + (4*b*c*Sqrt[(d + e*x)/(d + e/Sqr
t[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*Sqrt[1 + 1/(c^2
*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
+ e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
```

```
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !In
tegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] && !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{ce}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}\cdots}}}{ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{-c^2}\sqrt{\frac{d\cdots}{d-\cdots}}\right.}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \cdots$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \cdots$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \cdots$$

Mathematica [C] time = 4.95715, size = 307, normalized size = 1.08

$$\frac{2\left(ae(d+ex)-\frac{b\left(\frac{d}{x}+e\right)\left(-cex\operatorname{csch}^{-1}(cx)+\frac{\sqrt{2}\sqrt{1+icx}\left(cd(e+icd)\sqrt{-\frac{e(cx+i)}{cd-ie}}\sqrt{\frac{ce(cx+i)(d+ex)}{(e+icd)^2}}\Pi\left(\frac{icd}{e}+1;\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)-e^2(cx+i)\sqrt{\frac{c(d+ex)}{cd-ie}}\operatorname{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right.\right.\right.}{\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-\frac{e(cx+i)}{cd-ie}}(cd+cex)}}{c}\right.}{e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*e*(d + e*x) - (b*(e + d/x)*(-(c*e*x*ArcCsch[c*x]) + (Sqrt[2]*Sqrt[1 +
 I*c*x]*(-(e^2*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin[S
qrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]) + c*d*(I*c*d + e)*S
qrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)]))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[-((e*(I + c*x))/(c*d - I*e
))]*(c*d + c*e*x))))/c))/(e^2*Sqrt[d + e*x])
```

Maple [C] time = 0.281, size = 395, normalized size = 1.4

$$2\frac{1}{e}\left(a\sqrt{ex+d}+b\left(\sqrt{ex+d}\operatorname{arccsch}(cx)+2\frac{1}{cx}\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c}{c^2d^2+e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arccsch(c*x)+2/c*(-(I*(e*x+d)*c*e+(e*
x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-EllipticP
i((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)))
/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acsch}(cx)}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcsch}(cx) + a}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/sqrt(e*x + d), x)
```

3.61 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Rubi [A] time = 0.0767769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Mathematica [A] time = 6.30391, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Maple [A] time = 1.24, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x}\frac{1}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e*x^2 + d*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{x\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{\sqrt{ex + d}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x), x)
```

3.62 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi [A] time = 0.0821906, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Mathematica [A] time = 8.86307, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Maple [A] time = 5.133, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x^2} \frac{1}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(b\operatorname{arcsch}(cx) + a)}{ex^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{\sqrt{ex + d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

3.63 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=731

$$\frac{8bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4bc\sqrt{c^2x^2+1}\left(2c^2d^2-e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}}{15\left(-c^2\right)^{5/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^3*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsch[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/e^4 + (2*
(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) - (32*b*c*d*Sqrt[d + e*x]*Sqr
t[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*c*d^2*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)
^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*(2*c^2*d^2 - e^2
)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[
ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^
2]*e)])/(15*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b
*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)]
)/(5*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 2.63595, antiderivative size = 731, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {43, 6310, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(\right.}{5e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^3*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsch[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/e^4 + (2*
(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) - (32*b*c*d*Sqrt[d + e*x]*Sqr
t[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*c*d^2*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)
^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*(2*c^2*d^2 - e^2
)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[
ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^
2]*e)])/(15*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b
*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)]
)/(5*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right.}{e^4}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d}{}$$

$$= \frac{4b\sqrt{d+ex}\left(1+c^2x^2\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d}{}$$

Mathematica [C] time = 14.5785, size = 1042, normalized size = 1.43

$$\frac{a\left(\frac{ex}{d}+1\right)^{3/2}B_{-\frac{ex}{d}}\left(4,-\frac{1}{2}\right)d^4}{e^4(d+ex)^{3/2}}+\frac{b\left(-\frac{c^2\left(\frac{d}{x}+e\right)^2\left(\frac{2c^2\operatorname{csch}^{-1}(cx)d^2}{e^3\left(\frac{d}{x}+e\right)}-\frac{32c^2\operatorname{csch}^{-1}(cx)d^2}{5e^4}+\frac{32c\sqrt{1+\frac{1}{c^2x^2}}d}{15e^3}-\frac{2c^2x^2\operatorname{csch}^{-1}(cx)}{5e^2}-\frac{2cx\left(2e\sqrt{1+\frac{1}{c^2x^2}}-9cd\operatorname{csch}^{-1}(cx)\right)}{15e^3}\right)x^2}{(d+ex)^{3/2}}-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
 + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 + 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsch[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsch[c*x])/(e^3*(e + d/x))
- (2*c^2*x^2*ArcCsch[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 + 1/(c^2*x^2)] - 9*
c*d*ArcCsch[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(
3/2)*(-((Sqrt[2]*(32*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d +
 c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(
e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(48*c^3*d^3 - 8*c*d*
e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (16*c*d*e*Co
sh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*
I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((
e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(
c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellipti
cE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*
EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]
) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt
[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSi
n[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I
 + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2
 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

Maple [C] time = 0.305, size = 2019, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e^4*(a*(1/5*(e*x+d)^(5/2)-d*(e*x+d)^(3/2)+3*d^2*(e*x+d)^(1/2)+d^3/(e*x+d)
^(1/2))+b*(1/5*arccsch(c*x)*(e*x+d)^(5/2)-arccsch(c*x)*d*(e*x+d)^(3/2)+3*ar
ccsch(c*x)*d^2*(e*x+d)^(1/2)+arccsch(c*x)*d^3/(e*x+d)^(1/2)+2/15/c^3*(I*((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2)*(e*x+d)^(5/2)*c^3*d-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*
x+d)^(3/2)*c^2*d*e+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^3*d^
2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^2*e-24*(-(I*(e*x+d)*c
*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*
c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*
c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*
d^3-8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*
(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+
d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2))*c^3*d^3+32*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+48*(-(I*(e*x+d)*c*e+(e*x+d
)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(
1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3+I*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)*(e*x+d)^(1/2)*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*
c^3*d^3-I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*
((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((
e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2))*e^3+9*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e
-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*
d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-48*I*(-(I*(
e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-
(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c
```

```
/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c*d*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*
c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(
I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3 \operatorname{arcsch}(cx) + ax^3\right)\sqrt{ex+d}}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

3.64 $\int \frac{x^2\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=499

$$-\frac{20bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{2d^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\,\text{csch}\right.}{e^3}$$

```
[Out] (-2*d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
 b*ArcCsch[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) +
(4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]
*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^
2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
20*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*Ell
ipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - S
qrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) +
 (32*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2
]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d
 + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 2.00958, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {43, 6310, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424}

$$-\frac{2d^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{32bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}}{3ce^3x\sqrt{\frac{1}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```



```
[Out] (-2*d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
 b*ArcCsch[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) +
(4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]
*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^
2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
```

```
20*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*Ell
ipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - S
qrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) +
 (32*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2
]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d
 + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
```

```
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{2d^2 \left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

Mathematica [C] time = 14.3665, size = 979, normalized size = 1.96

$$b\left(\frac{2\left(\frac{d}{x}+e\right)^{3/2}(cx)^{3/2}\left(-\frac{5\sqrt{2}cde\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right),\frac{icd+e}{2e}\right)}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}+\frac{i\sqrt{2}(cd-ie)\left(8c^2d^2-e^2\right)\sqrt{icx+1}\sqrt{\frac{e(cx+i)(cd+cex)}{(icd+e)^2}}\,\Pi\left(\frac{icd}{e}+1;\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)}{e\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}+\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2
))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2) + (16*
c*d*ArcCsch[c*x])/(3*e^3) - (2*c*d*ArcCsch[c*x])/(e^2*(e + d/x)) - (2*c*x*A
rcCsch[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*((
-5*Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*
EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(
Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d
 + e)]) + (I*Sqrt[2]*(c*d - I*e)*(8*c^2*d^2 - e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*
(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sq
rt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x
^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x
)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*
x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*
d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d +
 c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x
)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x
]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*
d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e)
)]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1
+ 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2))))/(3*e^3*(d + e*x)^(3
/2))))/c^3
```

Maple [C] time = 0.299, size = 896, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e*
x+d)^(3/2)*arccsch(c*x)-2*arccsch(c*x)*d*(e*x+d)^(1/2)-arccsch(c*x)*d^2/(e*
x+d)^(1/2)-2/3/c^2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(5
*I*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-8*I*EllipticPi((e*x+d)^(1/2)*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^
2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c*d*e-4*EllipticF((e*x
+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+8*EllipticP
i((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*
c^2*d^2+EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE((e*x+d)^(1/2)*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e
^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^2}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

3.65 $\int \frac{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=318

$$\frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

```
[Out] (2*d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Ar
cCsch[c*x]))/e^2 + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt
[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c
^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqr
t[d + e*x]) - (8*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1
 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sq
rt[-c^2]*d + e)])/(c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 1.69131, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {43, 6310, 12, 6721, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{2\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}-\frac{8bd\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Ar
cCsch[c*x]))/e^2 + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt
[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c
^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqr
t[d + e*x]) - (8*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1
 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sq
rt[-c^2]*d + e)])/(c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{(2b)\int \frac{2d+ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce^2}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{2d+ex}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(\frac{e}{\sqrt{d+ex}\sqrt{1+c^2x^2}} + \cdots\right.}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-\cdots}}}}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}F\left(\text{si}\cdots\right.}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}F\left(\text{si}\cdots\right.}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}F\left(\text{si}\cdots\right.}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x}$$

Mathematica [C] time = 1.55801, size = 264, normalized size = 0.83

$$\frac{2\left(-\frac{2ib\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right),\frac{cd-ie}{cd+ie}\right)-2\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{cx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-\frac{c}{cd-ie}}}+\frac{a(2d+ex)}{\sqrt{d+ex}}+\frac{b\text{csch}^{-1}(cx)(2d+ex)}{\sqrt{d+ex}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsch[c*x])/Sqrt[d + e
*x] - ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*
d - I*e))]*(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d
 - I*e)/(c*d + I*e)] - 2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*
d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c*Sqrt[-(c/(c*d - I*
e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/e^2
```

Maple [C] time = 0.323, size = 418, normalized size = 1.3

$$2\frac{1}{e^2}\left(a\left(\sqrt{ex+d}+\frac{d}{\sqrt{ex+d}}\right)+b\left(\sqrt{ex+d}\,\text{arccsch}\,(cx)+\frac{\text{arccsch}\,(cx)\,d}{\sqrt{ex+d}}+2\frac{1}{cx}\sqrt{-\frac{i\,(ex+d)\,ce+(ex+d)\,c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e^2*(a*((e*x+d)^(1/2)+d/(e*x+d)^(1/2))+b*((e*x+d)^(1/2)*arccsch(c*x)+arcc
sch(c*x)*d/(e*x+d)^(1/2)+2/c*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*(EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c
*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*
d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)))/(((e*x+d)^2*c^2-2*(e*x+
d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(3/2), x)
```

3.66 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

```
[Out] (-2*(a + b*ArcCsch[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*
x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*
x*Sqrt[d + e*x])
```

Rubi [A] time = 0.284051, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6290, 1574, 933, 168, 538, 537}

$$\frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCsch[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*
x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*
x*Sqrt[d + e*x])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx &= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\, d\ldots\right.}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{4b\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] time = 0.655283, size = 166, normalized size = 1.11

$$
\frac{-2e\left(c^2x^2+1\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)+2bcx\sqrt{\frac{2}{c^2x^2}+2}\sqrt{1+icx}(e+icd)\sqrt{\frac{ce(cx+i)(d+ex)}{(e+icd)^2}}\,\Pi\left(\frac{icd}{e}+1;\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e^2\left(c^2x^2+1\right)\sqrt{d+ex}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*e*(1 + c^2*x^2)*(a + b*ArcCsch[c*x]) + 2*b*c*(I*c*d + e)*Sqrt[2 + 2/(c^
2*x^2)]*x*Sqrt[1 + I*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*Ell
```

```
ipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d +
 e)/(2*e)])/(e^2*Sqrt[d + e*x]*(1 + c^2*x^2))
```

Maple [C] time = 0.299, size = 328, normalized size = 2.2

$$2\frac{1}{e}\left(-\frac{a}{\sqrt{ex+d}}+b\left(-\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}}+2\frac{1}{cdx}\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c^2d^2}{c^2d^2+e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsch(c*x)+2/c/(((e*x+d)^2*c^2-
2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/d/((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2)*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)
*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi
((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2
)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{e^2x^2+2dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acsch}(cx)}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(3/2), x)
```

3.67 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi [A] time = 0.0901345, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Mathematica [A] time = 11.4617, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Maple [A] time = 2.531, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x}\,(ex+d)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex+d}(b\,\mathrm{arcsch}\,(cx)+a)}{e^2x^3+2\,dex^2+d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

3.68 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi [A] time = 0.0989249, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Mathematica [A] time = 14.5677, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Maple [A] time = 6.172, size = 0, normalized size = 0.

$$\int \frac{a + b\text{arccsch}\left(cx\right)}{x^{2}} \left(ex + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(b\,\text{arcsch}\left(cx\right) + a)}{e^{2}x^{4} + 2\,dex^{3} + d^{2}x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

3.69 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=777

$$-\frac{32bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}$$

```
[Out] (4*b*d^2*(1 + c^2*x^2))/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
rt[d + e*x]) + (2*d^3*(a + b*ArcCsch[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*Arc
Csch[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^4) - (8*b*S
qrt[-c^2]*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*e^3*(c
^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c
^2]*e)]) + (4*b*c*(2*c^2*d^2 + e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[
-c^2]*e)])/(3*(-c^2)^(3/2)*e^3*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (32*b*c*d*Sqrt[(c^2*(d + e*x))/
(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)
*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (64*b*d^2*Sqrt[(Sqrt[-c^2]*(d
 + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^4*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 3.18554, antiderivative size = 777, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 18, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {43, 6310, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419, 1651}

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}+\frac{\cdots}{3ce^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*d^2*(1 + c^2*x^2))/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
rt[d + e*x]) + (2*d^3*(a + b*ArcCsch[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*Arc
Csch[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^4) - (8*b*S
qrt[-c^2]*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*e^3*(c
^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c
^2]*e)]) + (4*b*c*(2*c^2*d^2 + e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[
-c^2]*e)])/(3*(-c^2)^(3/2)*e^3*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (32*b*c*d*Sqrt[(c^2*(d + e*x))/
(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)
*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (64*b*d^2*Sqrt[(Sqrt[-c^2]*(d
 + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^4*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
```

```
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+\cdots}{}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+\cdots}{}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+\cdots}{}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+\cdots}{}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+\cdots}{}$$

$$= \frac{68bd^2\left(1+c^2x^2\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \cdots$$

$$= \frac{68bd^2\left(1+c^2x^2\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \cdots$$

$$= \frac{4bd^2\left(1+c^2x^2\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \cdots$$

$$= \frac{4bd^2\left(1+c^2x^2\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \cdots$$

$$= \frac{4bd^2\left(1+c^2x^2\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \cdots$$

$$= \frac{4bd^2\left(1+c^2x^2\right)}{\cdots} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\cdots}\;\frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\cdots}$$

<u>Mathematica</u> [C] time = 14.3462, size = 1108, normalized size = 1.43

$$\frac{a\left(\frac{ex}{d}+1\right)^{5/2} B_{-\frac{ex}{d}}\left(4,-\frac{3}{2}\right) d^4}{e^4(d+ex)^{5/2}} + \frac{b\Bigg(2\left(\frac{d}{x}+e\right)^{5/2}(cx)^{5/2}\Bigg(\frac{2\cosh\left(2\operatorname{csch}^{-1}(cx)\right)\Bigg(\frac{cx\Big(cd\sqrt{2icx+2}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\,\text{EllipticF}\Big(\sin^{-1}\Big(\sqrt{-\frac{e(cx+i)}{cd-ie}}\Big),\frac{icd+e}{2e}\Big)+2\sqrt{-\frac{e(cx-i)}{cd+ie}}(cx+i)\sqrt{\frac{cd-\cdots}{cd\cdots}}\cdots}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
 + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e*(c^2*d^2 + e
^2)) + (32*c*d*ArcCsch[c*x])/(3*e^4) - (2*c*d*ArcCsch[c*x])/(3*e^2*(e + d/x
)^2) - (2*c*x*ArcCsch[c*x])/(3*e^3) - (2*(2*c^2*d^2*e*Sqrt[1 + 1/(c^2*x^2)]
 + 7*c^3*d^3*ArcCsch[c*x] + 7*c*d*e^2*ArcCsch[c*x]))/(3*e^3*(c^2*d^2 + e^2)
*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]
*(8*c^3*d^3*e + 8*c*d*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*
d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*
x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(16*c^4*d^4 + 16*c^2*d^2*e^2 -
e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e^3*Cosh[
2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*
c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[
ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*Ell
ipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) +
 (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e
*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[S
qrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I +
c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 +
c^2*x^2))))/(3*e^4*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c^4
```

Maple [C] time = 0.316, size = 2726, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^4*(a*(1/3*(e*x+d)^(3/2)-3*d*(e*x+d)^(1/2)-3*d^2/(e*x+d)^(1/2)+1/3*d^3/(
e*x+d)^(3/2))+b*(1/3*(e*x+d)^(3/2)*arccsch(c*x)-3*arccsch(c*x)*d*(e*x+d)^(1
/2)-3*arccsch(c*x)*d^2/(e*x+d)^(1/2)+1/3*arccsch(c*x)*d^3/(e*x+d)^(3/2)+2/3
/c^2*(16*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)
*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi
((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2
)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(
e*x+d)^(1/2)*c^3*d^3*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^4*d^3+
I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^3-2*I*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)*(e*x+d)*c^3*d^3*e+8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^4*d^4-16*(-(I*(e*x+d
)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+
d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2
*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^4*d^4-8
*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*
x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^
(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^3+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x
+d)*c^4*d^4+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4*e-((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2)*c^4*d^5+16*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(
c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+
c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
*(e*x+d)^2*c^3*d^2*e+7*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e^2+(-(I*(e*x+d)*c*e
+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^
2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d
)^(1/2)*c^2*d^2*e^2-16*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2
```

```
+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*
(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e^2-8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3*e-
((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e^2-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*
d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*e^4+(
-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)
*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2
)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2))*(e*x+d)^(1/2)*e^4)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^
2/x^2/e^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3\,\text{arcsch}\,(cx) + ax^3\right)\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2
+ 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

3.70 $\int \frac{x^2\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=569

$$\frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{2d^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\ldots$$

```
[Out] (-4*b*d*(1 + c^2*x^2))/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a
+ b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*
x]))/e^3 + (4*b*Sqrt[-c^2]*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e
)])/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(
c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 2.49705, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.81, Rules used = {43, 6310, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419}

$$-\frac{2d^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4bd\left(c^2x^2+1\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```


```
[Out] (-4*b*d*(1 + c^2*x^2))/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a
+ b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*
```

```
x]))/e^3 + (4*b*Sqrt[-c^2]*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e
)])/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(
c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{b\int \frac{2(}{3e^3}}{}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{(2b)\int}{}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(2b\sqrt{1}\right.}{}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(2b\sqrt{1}\right.}{}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(16bd^2\right.}{}$$

$$= -\frac{12bd\left(1+c^2x^2\right)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2}{}$$

$$= -\frac{12bd\left(1+c^2x^2\right)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2}{}$$

$$= -\frac{4bd\left(1+c^2x^2\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= -\frac{4bd\left(1+c^2x^2\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= -\frac{4bd\left(1+c^2x^2\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= -\frac{4bd\left(1+c^2x^2\right)}{} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{} +$$

Mathematica [C] time = 14.2712, size = 1076, normalized size = 1.89

$$b\left(-\frac{c^3\left(\frac{d}{x}+e\right)^3\left(-\frac{4c\sqrt{1+\frac{1}{c^2x^2}}\,d}{3e^2\left(c^2d^2+e^2\right)}+\frac{2\operatorname{csch}^{-1}(cx)}{3e\left(\frac{d}{x}+e\right)^2}-\frac{16\operatorname{csch}^{-1}(cx)}{3e^3}+\frac{4\left(2c^2\operatorname{csch}^{-1}(cx)d^2+ce\sqrt{1+\frac{1}{c^2x^2}}\,d+2e^2\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(c^2d^2+e^2\right)\left(\frac{d}{x}+e\right)}\right)x^3}{(d+ex)^{5/2}}-\frac{2\left(\frac{d}{x}+e\right)^{5/2}(cx)^{5/2}\left(-\frac{\sqrt{2}\left(3e^3+3c^2d^2e\right)\sqrt{icx+1}\,(cx+i)\sqrt{\frac{cd+c}{cd-}}}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+}}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2
))) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2*(c^
2*d^2 + e^2)) - (16*ArcCsch[c*x])/(3*e^3) + (2*ArcCsch[c*x])/(3*e*(e + d/x)
^2) + (4*(c*d*e*Sqrt[1 + 1/(c^2*x^2)] + 2*c^2*d^2*ArcCsch[c*x] + 2*e^2*ArcC
sch[c*x]))/(3*e^2*(c^2*d^2 + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e + d
/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]*(3*c^2*d^2*e + 3*e^3)*Sqrt[1 + I*c*x]*(I
+ c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x)
)/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*
(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(8
*c^3*d^3 + 9*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c
*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e
))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2
)) - (2*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(
c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF
[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-(
(e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*
d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*
d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I
*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c
*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
(I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))
/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(2 + c^2*x^2))))/(3*e^3*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c^
3
```

Maple [C] time = 0.319, size = 2497, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)+2*d/(e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2))+b*((e*x+d)
^(1/2)*arccsch(c*x)+2*arccsch(c*x)*d/(e*x+d)^(1/2)-1/3*arccsch(c*x)*d^2/(e*
x+d)^(3/2)-2/3/c*(-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e-((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2)*(e*x+d)^2*c^3*d^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+
d)^2*c^2*d*e+4*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(
1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3-(-(I*(e*x+d)*c*e+(e*x+d)*c^2
*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*
d^3+8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((
I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e
*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d
,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x
+d)^(1/2)*e^3-8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^
(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellip
ticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^
2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2))*(e*x+d)^(1/2)*c^3*d^3-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(
c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c
*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*e^3+2*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2)*(e*x+d)*c^3*d^3+8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d
^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2)*c^3*d^4+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e+4*(-(I*(e*x
+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*
x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
```

```
*(e*x+d)^(1/2)*c*d*e^2-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-2*I*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2)*(e*x+d)*c^2*d^2*e-8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
,1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)*d*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^2)/(((e*x+d)^2*c^2
-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1
/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

3.71 $\int \frac{x\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=393

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}+\frac{4b\left(c^2x^2+1\right)}{3cx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}-\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\sin^{-1}\right.}{3cex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right.}$$

```
[Out] (4*b*(1 + c^2*x^2))/(3*c*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x]) + (2*d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
ch[c*x]))/(e^2*Sqrt[d + e*x]) - (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c
^2*d - Sqrt[-c^2]*e)])/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x])
```

Rubi [A] time = 2.19119, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {43, 6310, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537}

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}+\frac{4b\left(c^2x^2+1\right)}{3cx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}-\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\sin^{-1}\right.}{3cex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*(1 + c^2*x^2))/(3*c*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x]) + (2*d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
ch[c*x]))/(e^2*Sqrt[d + e*x]) - (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c
^2*d - Sqrt[-c^2]*e)])/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
```

```
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{b\int \frac{2(-2d-3ex)}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{(2b)\int \frac{-2d-3ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^2}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1+c^2x^2}} - \frac{2\ldots}{x(d+ex)^{3/2}\ldots}\right.}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{\ldots}\right.}{}$$

$$= \frac{4b\left(1+c^2x^2\right)}{c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1+\ldots}\right.}{}$$

$$= \frac{4b\left(1+c^2x^2\right)}{c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4b\sqrt{1+\ldots}\right.}{}$$

$$= \frac{4b\left(1+c^2x^2\right)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4b\sqrt{1\ldots}\right.}{}$$

$$= \frac{4b\left(1+c^2x^2\right)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{4b\sqrt{-c^2\ldots}}{}$$

$$= \frac{4b\left(1+c^2x^2\right)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{4b\sqrt{-c^2\ldots}}{}$$

$$= \frac{4b\left(1+c^2x^2\right)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{\ldots}}x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{4b\sqrt{-c^2\ldots}}{}$$

Mathematica [C] time = 2.41961, size = 390, normalized size = 0.99

$$\frac{2}{3}\left(\frac{2ib\sqrt{-\frac{c}{cd-ie}}\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}\left(-cd\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right),\frac{cd-ie}{cd+ie}\right)+cdE\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)\right.\right.}{c^2de^2x\sqrt{\frac{1}{c^2x^2}+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/((c^2*d^2 + e^2)*Sqrt[d + e*x]) - (a*(2
*d + 3*e*x))/(e^2*(d + e*x)^(3/2)) - (b*(2*d + 3*e*x)*ArcCsch[c*x])/(e^2*(d
 + e*x)^(3/2)) + ((2*I)*b*Sqrt[-(c/(c*d - I*e))]*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(c*d*EllipticE[I*ArcSinh[Sqrt[
-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - c*d*EllipticF[
I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] +
 2*(c*d - I*e)*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]
*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^2*d*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x)))/3
```

Maple [C] time = 0.308, size = 2107, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^2*(a*(-1/(e*x+d)^(1/2)+1/3*d/(e*x+d)^(3/2))+b*(-1/(e*x+d)^(1/2)*arccsch
(c*x)+1/3*arccsch(c*x)*d/(e*x+d)^(3/2)+2/3/c*(I*((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2)*c^2*d^3*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^3*d^2+(-(I*(
e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-
(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2))*(e*x+d)^(1/2)*c^3*d^3-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e
-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3+2*I*(-(I*(e*x+d)*
c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)
*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d
```

```
^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e-2
*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+
d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(
1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e
-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/
2)*c^3*d^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^2*d*e+2*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^3*d^3+2*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*
d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/(
(I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*e^3-((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2)*c^3*d^4+(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*El
lipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-(-(I*(e*x+d)*c*e+(e*x+d)
*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*
c*d*e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*d*e^3-2*(-(I*(e*x+d)*c*e+(e*x+d
)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(
1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-2*I*((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
)*c*d^2*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2
)/x/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)
/d))
```

<u>Maxima</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx\,\text{arcsch}\,(cx)+ax)\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*
d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(5/2), x)
```

3.72 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=369

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}-\frac{4be\left(c^2x^2+1\right)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}+\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right.}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}$$

```
[Out] (-4*b*e*(1 + c^2*x^2))/(3*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(3*e*(d + e*x)^(3/2)) + (4*b*Sqrt[-c^2
]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/S
qrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*d*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*Sqrt[(Sq
rt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*d*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 0.566642, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6290, 1574, 958, 745, 21, 719, 424, 933, 168, 538, 537}

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}-\frac{4be\left(c^2x^2+1\right)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}+\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right.}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*e*(1 + c^2*x^2))/(3*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(3*e*(d + e*x)^(3/2)) + (4*b*Sqrt[-c^2
]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/S
qrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*d*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*Sqrt[(Sq
rt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*d*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
```

```
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3cde\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(4bc\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,d}{3d\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2bc\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3d\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} + \cdots$$

$$= -\frac{4be\left(1+c^2x^2\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\text{Su}\cdots}{3cd\left(c^2d^2+e^2\right)\cdots}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\text{si}\cdots\right.}{3cd\left(c^2d^2+e^2\right)\sqrt{1\cdots}}$$

Mathematica [C] time = 14.0635, size = 892, normalized size = 2.42

$$b\left(\frac{2\left(\frac{d}{x}+e\right)^{5/2}(cx)^{5/2}\left(\frac{i\sqrt{2}cd(cd-ie)\sqrt{icx+1}\sqrt{\frac{e(cx+i)(cd+cex)}{(icd+e)^2}}\Pi\left(\frac{icd}{e}+1;\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}+\frac{2e\cosh\left(2\text{csch}^{-1}(cx)\right)\left(cx\left(cd\sqrt{2icx+2}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right),\ldots\right.\right.\right.}{\ldots}\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-2*a)/(3*e*(d + e*x)^(3/2)) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/(
c^2*x^2)])/(3*c*d*(c^2*d^2 + e^2)) + (2*ArcCsch[c*x])/(3*c^2*d^2*e) + (2*e*
ArcCsch[c*x])/(3*c^2*d^2*(e + d/x)^2) - (4*(-(c*d*e*Sqrt[1 + 1/(c^2*x^2)])
+ c^2*d^2*ArcCsch[c*x] + e^2*ArcCsch[c*x]))/(3*c^2*d^2*(c^2*d^2 + e^2)*(e +
 d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((I*Sqrt[2]*c*d*
(c*d - I*e)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]
*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c
*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e*
Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (
2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))
/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellip
ticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*
e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*
(I + c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2))))/(3*e*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c
```

Maple [C] time = 0.316, size = 2079, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsch(c*x)-2/3/c*(-I*(-(I
*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*
e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*
((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)
*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^
2*d^2*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^3*d^2+(-(I*(e*x+d)*c*
e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c
^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+
d)^(1/2)*c^3*d^3-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))
^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Elli
pticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3-2*I*((I*e+c*d)*c/(c^2*d^2+
e^2))^(1/2)*(e*x+d)*c^2*d^2*e+(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(
c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+
c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3-I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^
2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(
1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*e^3+2*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)*(e*x+d)*c^3*d^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e-((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4+(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),
(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-(-(I*
(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e
-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2))*(e*x+d)^(1/2)*c*d*e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*d*e^3+(-(I*(
e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-
(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c
/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*
e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^2*d*e-((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2)*c*d^2*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^
2/x^2/e^2)^(1/2)/x/d^2/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(5/2), x)
```

3.73 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi [A] time = 0.0938044, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Mathematica [A] time = 30.2035, size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Maple [A] time = 4.395, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x} (ex + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(b\operatorname{arcsch}(cx) + a)}{e^3x^4 + 3\,de^2x^3 + 3\,d^2ex^2 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*
e*x^2 + d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{5}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

3.74 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi [A] time = 0.106008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Mathematica [A] time = 29.2403, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Maple [A] time = 7.439, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^2}(ex + d)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex + d}(b\,\mathrm{arcsch}\,(cx) + a)}{e^3x^5 + 3\,de^2x^4 + 3\,d^2ex^3 + d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*
e*x^3 + d^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{5}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

3.75 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=648

$$-\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}-\frac{2\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-\frac{4be\left(c^2x^2+1\right)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e\right.}$$

```
[Out] (-4*b*e*(1 + c^2*x^2))/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
 e*x)^(3/2)) - (16*b*c*e*(1 + c^2*x^2))/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*e*(1 + c^2*x^2))/(5*c*d^2*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(5*e*(d +
 e*x)^(5/2)) - (4*b*c*(7*c^2*d^2 + 3*e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*E
llipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*Sqrt[-c^2]*e)/(-(c^2*d)
 + Sqrt[-c^2]*e)])/(15*Sqrt[-c^2]*d^2*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c^2*x^2
)]*x*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]) - (4*b*Sqrt[-c^2]*Sqrt[(d + e*x)/(
d + e/Sqrt[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*c*d*(c^2*d^2 + e
^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*x
))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt
[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(5*c*d^2*e*Sqrt[1 + 1/(c^2*x
^2)]*x*Sqrt[d + e*x])
```

Rubi [A] time = 1.03544, antiderivative size = 785, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6290, 1574, 958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$-\frac{2\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-\frac{4be\left(c^2x^2+1\right)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}-\frac{16bce\left(c^2x^2+1\right)}{15x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)^2\sqrt{d+ex}}-\frac{4be\left(c^2x\right.}{15cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
```


```
[Out] (-4*b*e*(1 + c^2*x^2))/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
 e*x)^(3/2)) - (16*b*c*e*(1 + c^2*x^2))/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*e*(1 + c^2*x^2))/(5*c*d^2*(c^2*d^2 + e^2)*
```

```
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(5*e*(d +
 e*x)^(5/2)) + (16*b*c*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE
[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c
^2]*e)])/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d +
e/Sqrt[-c^2])]) + (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE
[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c
^2]*e)])/(5*c*d^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d
 + e/Sqrt[-c^2])]) - (4*b*Sqrt[-c^2]*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]*Sqr
t[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]
*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2
*e)/(Sqrt[-c^2]*d + e)])/(5*c*d^2*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
```

```
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}\,dx}{5ce}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}} + \frac{1}{d^2x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right.}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5cd\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{4be\left(1+c^2x^2\right)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\text{csch}^{-1}\right.}{5e(d+ex)}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce\left(1+c^2x^2\right)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4be}{5cd^2\left(c^2d^2+e^2\right.}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce\left(1+c^2x^2\right)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4be}{5cd^2\left(c^2d^2+e^2\right.}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce\left(1+c^2x^2\right)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4be}{5cd^2\left(c^2d^2+e^2\right.}$$

$$= -\frac{4be\left(1+c^2x^2\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce\left(1+c^2x^2\right)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4be}{5cd^2\left(c^2d^2+e^2\right.}$$

Mathematica [C] time = 14.2953, size = 1217, normalized size = 1.88

$$b\left(\frac{2\left(\frac{d}{x}+e\right)^{7/2}(cx)^{7/2}\left(-\frac{\sqrt{2}\left(e^3+c^2d^2e\right)\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right),\frac{icd+e}{2e}\right)}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}\,(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}+\frac{i\sqrt{2}(cd-ie)\left(3c^3d^3-cde^2\right)\sqrt{icx+1}\sqrt{\frac{e(cx+i)(cd+cex)}{(icd+e)^2}}\,\Pi\left(\frac{icd}{e}+1;\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)\middle|\ldots\right.}{e\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}\,(cx)^{3/2}}\ldots\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((-4*(7*c^2*d^2 +
 3*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(15*c^2*d^2*(c^2*d^2 + e^2)^2) + (2*ArcCsch[
c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsch[c*x])/(5*c^3*d^3*(e + d/x)^3) + (2*(-2
*c*d*e^2*Sqrt[1 + 1/(c^2*x^2)] + 9*c^2*d^2*e*ArcCsch[c*x] + 9*e^3*ArcCsch[c
*x]))/(15*c^3*d^3*(c^2*d^2 + e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 +
 1/(c^2*x^2)] - 8*c*d*e^3*Sqrt[1 + 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsch[c*x] +
18*c^2*d^2*e^2*ArcCsch[c*x] + 9*e^4*ArcCsch[c*x]))/(15*c^3*d^3*(c^2*d^2 + e
^2)^2*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*(-((Sq
rt[2]*(c^2*d^2*e + e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d -
 I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2
*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))
/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(3*c^3*d^3 - c*d*e^2)*Sqrt[1 + I*c
*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/
e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[
1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-7*c^2*d^2*e - 3*e^3)*Cos
h[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I
)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e
*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c
*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Elliptic
E[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*E
llipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)])
 + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[
(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin
[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I
```

```
+ c*x))/(c*d - I*e))]))))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]
*(2 + c^2*x^2))))/(15*c*d*e*(c^2*d^2 + e^2)^2*(d + e*x)^(7/2))))/c
```

Maple [C] time = 0.326, size = 3782, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(7/2),x)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsch(c*x)-2/15/c*(I*(-(I
*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*
e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*(
(I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2))*(e*x+d)^(3/2)*c^4*d^4*e-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^
2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(
I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^4*d^4*e-6*I*(-(I*(e*x+d)*c*e+(e*x+d)*
c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/
2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^2*d^2*e^3+I*(-(I*(e*x
+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*
x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*(e*x+d)^(3/2)*c^2*d^2*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^5*d^7-5*I*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^2*d^2*e^3+8*I*((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2)*(e*x+d)*c^2*d^3*e^3-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1
/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*e^5+6*(-(I*(e*x+d)*c*e+(e*x+d)*c
^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^
5*d^5-7*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((
I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*
x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^5*d^5+3*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^
2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/
```

```
(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(
1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^5*d^5+7*I*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2)*(e*x+d)^3*c^4*d^3*e+5*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e
*x+d)*c^4*d^5*e-13*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^4*d^4*e+
3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^3*c^2*d*e^3-7*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2)*(e*x+d)^3*c^5*d^4+13*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(
e*x+d)^2*c^5*d^5-5*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^5*d^6-2*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^5*e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2)*d^2*e^5-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^3*e^4+5*((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2)*(e*x+d)^2*c^3*d^3*e^2-8*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(
e*x+d)*c^3*d^4*e^2-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c*d^2*e^4+3*
I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*d*e^5-3*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2)*(e*x+d)^3*c^3*d^2*e^2+2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*
d^4*e^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^6*e+9*(-(I*(e*x+d)*c*e+(e
*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d
+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(
3/2)*c^3*d^3*e^2-10*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^
2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*E
llipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^3*d^3*e^2+6*(-(I*(e*x+d)*c*e+
(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2
*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e
^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c^3*d^3*e^2+3*(
-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)
*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2
)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2))*(e*x+d)^(3/2)*c*d*e^4-3*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c*d*e^4+3*(-(I*(e*
x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e
*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(
c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(3/2)*c*d*e^
4)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/d^3/(c
^2*d^2+e^2)^2/(e*x+d)^(3/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*
e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(7/2), x)
```

3.76 $\int x^4 \left(d+ex^2\right) \left(a+b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=214

$$\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^4\sqrt{-c^2x^2-1}\left(42c^2d-25e\right)}{840c^3\sqrt{-c^2x^2}}-\frac{bx^2\sqrt{-c^2x^2-1}\left(42c^2d-25e\right)}{560c^5\sqrt{-c^2x^2}}-\frac{bx\left(\right.}{}$$

```
[Out] -(b*(42*c^2*d - 25*e)*x^2*Sqrt[-1 - c^2*x^2])/(560*c^5*Sqrt[-(c^2*x^2)]) +
(b*(42*c^2*d - 25*e)*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*Sqrt[-(c^2*x^2)]) + (
b*e*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)]) + (d*x^5*(a + b*ArcCsch
[c*x]))/5 + (e*x^7*(a + b*ArcCsch[c*x]))/7 - (b*(42*c^2*d - 25*e)*x*ArcTan[
(c*x)/Sqrt[-1 - c^2*x^2]])/(560*c^6*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.132, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {14, 6302, 12, 459, 321, 217, 203}

$$\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^4\sqrt{-c^2x^2-1}\left(42c^2d-25e\right)}{840c^3\sqrt{-c^2x^2}}-\frac{bx^2\sqrt{-c^2x^2-1}\left(42c^2d-25e\right)}{560c^5\sqrt{-c^2x^2}}-\frac{bx\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```


```
[Out] -(b*(42*c^2*d - 25*e)*x^2*Sqrt[-1 - c^2*x^2])/(560*c^5*Sqrt[-(c^2*x^2)]) +
(b*(42*c^2*d - 25*e)*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*Sqrt[-(c^2*x^2)]) + (
b*e*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)]) + (d*x^5*(a + b*ArcCsch
[c*x]))/5 + (e*x^7*(a + b*ArcCsch[c*x]))/7 - (b*(42*c^2*d - 25*e)*x*ArcTan[
(c*x)/Sqrt[-1 - c^2*x^2]])/(560*c^6*Sqrt[-(c^2*x^2)])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^4 \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int \frac{x^4\left(7d+5ex^2\right)}{35\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int \frac{x^4\left(7d+5ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{35\sqrt{-c^2x^2}}$$

$$= \frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{\left(bc\left(42d\right.\right.}{}$$

$$= \frac{b\left(42c^2d-25e\right)x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e$$

$$= -\frac{b\left(42c^2d-25e\right)x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b\left(42c^2d-25e\right)x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2}}{42c\sqrt{-c^2x^2}}$$

$$= -\frac{b\left(42c^2d-25e\right)x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b\left(42c^2d-25e\right)x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2}}{42c\sqrt{-c^2x^2}}$$

$$= -\frac{b\left(42c^2d-25e\right)x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b\left(42c^2d-25e\right)x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2}}{42c\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.232764, size = 138, normalized size = 0.64

$$\frac{48ac^7x^5\left(7d+5ex^2\right)+bc^2x^2\sqrt{\frac{1}{c^2x^2}+1}\left(c^4\left(84dx^2+40ex^4\right)-2c^2\left(63d+25ex^2\right)+75e\right)+3b\left(42c^2d-25e\right)\log\left(x\left(\sqrt{\frac{1}{c^2x^2}}\right.\right.}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2*(75*e - 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
*x^2)*ArcCsch[c*x] + 3*b*(42*c^2*d - 25*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*
x])/(1680*c^7)
```

Maple [A] time = 0.196, size = 211, normalized size = 1.

$$\frac{1}{c^5}\left(\frac{a}{c^2}\left(\frac{ec^7x^7}{7}+\frac{c^7x^5d}{5}\right)+\frac{b}{c^2}\left(\frac{\operatorname{arccsch}(cx)\,ec^7x^7}{7}+\frac{\operatorname{arccsch}(cx)\,c^7x^5d}{5}+\frac{1}{1680\,cx}\sqrt{c^2x^2+1}\left(40\,ec^5x^5\sqrt{c^2x^2+1}+84\,c^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^5*(a/c^2*(1/7*e*c^7*x^7+1/5*c^7*x^5*d)+b/c^2*(1/7*arccsch(c*x)*e*c^7*x^
7+1/5*arccsch(c*x)*c^7*x^5*d+1/1680*(c^2*x^2+1)^(1/2)*(40*e*c^5*x^5*(c^2*x^
2+1)^(1/2)+84*c^5*d*x^3*(c^2*x^2+1)^(1/2)-50*e*c^3*x^3*(c^2*x^2+1)^(1/2)-12
6*c^3*d*x*(c^2*x^2+1)^(1/2)+126*c^2*d*arcsinh(c*x)+75*e*c*x*(c^2*x^2+1)^(1/
2)-75*e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.0222, size = 390, normalized size = 1.82

$$\frac{1}{7}aex^7+\frac{1}{5}adx^5+\frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd+\frac{1}{672}\left(96\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(
c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/
(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c
^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))
/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2)
+ 1) - c^6) - 15*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^
2) + 1) - 1)/c^6)/c)*b*e
```

Fricas [A] time = 3.32047, size = 680, normalized size = 3.18

$$\frac{240\,ac^7ex^7 + 336\,ac^7dx^5 + 48\left(7\,bc^7d + 5\,bc^7e\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 3\left(42\,bc^2d - 25\,be\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - 48}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(7*b*c^7*d + 5*b*c^7*e)*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 3*(42*b*c^2*d - 25*b*e)*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 48*(7*b*c^7*d + 5*b*c^7*e)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d
*x^5 - 7*b*c^7*d - 5*b*c^7*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + (40*b*c^6*e*x^6 + 2*(42*b*c^6*d - 25*b*c^4*e)*x^4 - 3*(42*b*c^4*d -
 25*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(a + b\,\mathrm{acsch}\,(cx)\right)\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)\left(b\,\mathrm{arcsch}\,(cx) + a\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^4, x)
```

3.77 $\int x^2 \left(d+ex^2\right) \left(a+b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=167

$$\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{-c^2x^2-1}\left(20c^2d-9e\right)}{120c^3\sqrt{-c^2x^2}}+\frac{bx\left(20c^2d-9e\right)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{120c^4\sqrt{-c^2x^2}}+$$

```
[Out] (b*(20*c^2*d - 9*e)*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) + (b
*e*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + (d*x^3*(a + b*ArcCsch[
c*x]))/3 + (e*x^5*(a + b*ArcCsch[c*x]))/5 + (b*(20*c^2*d - 9*e)*x*ArcTan[(c
*x)/Sqrt[-1 - c^2*x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.108677, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {14, 6302, 12, 459, 321, 217, 203}

$$\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{-c^2x^2-1}\left(20c^2d-9e\right)}{120c^3\sqrt{-c^2x^2}}+\frac{bx\left(20c^2d-9e\right)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{120c^4\sqrt{-c^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(20*c^2*d - 9*e)*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) + (b
*e*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + (d*x^3*(a + b*ArcCsch[
c*x]))/3 + (e*x^5*(a + b*ArcCsch[c*x]))/5 + (b*(20*c^2*d - 9*e)*x*ArcTan[(c
*x)/Sqrt[-1 - c^2*x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{15\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$=\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{15\sqrt{-c^2x^2}}$$

$$=\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{\left(bc\left(20\right.\right.}{}$$

$$=\frac{b\left(20c^2d-9e\right)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e$$

$$=\frac{b\left(20c^2d-9e\right)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e$$

$$=\frac{b\left(20c^2d-9e\right)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e$$

Mathematica [A] time = 0.166842, size = 119, normalized size = 0.71

$$\frac{c^2x^2\left(8ac^3x\left(5d+3ex^2\right)+b\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(20d+6ex^2\right)-9e\right)\right)+b\left(9e-20c^2d\right)\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)\right)+8bc^5x^3\text{csch}^{-1}(}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-9*e + c^2*(
20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsch[c*x] + b*(-20*c^2*d
 + 9*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] time = 0.18, size = 171, normalized size = 1.

$$\frac{1}{c^3}\left(\frac{a}{c^2}\left(\frac{c^5x^5e}{5}+\frac{c^5x^3d}{3}\right)+\frac{b}{c^2}\left(\frac{\text{arccsch}\,(cx)\,c^5x^5e}{5}+\frac{\text{arccsch}\,(cx)\,c^5x^3d}{3}-\frac{1}{120\,cx}\sqrt{c^2x^2+1}\left(-6\,ec^3x^3\sqrt{c^2x^2+1}-20\,c^3d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^3*(a/c^2*(1/5*c^5*x^5*e+1/3*c^5*x^3*d)+b/c^2*(1/5*arccsch(c*x)*c^5*x^5*
e+1/3*arccsch(c*x)*c^5*x^3*d-1/120*(c^2*x^2+1)^(1/2)*(-6*e*c^3*x^3*(c^2*x^2
+1)^(1/2)-20*c^3*d*x*(c^2*x^2+1)^(1/2)+20*c^2*d*arcsinh(c*x)+9*e*c*x*(c^2*x
^2+1)^(1/2)-9*e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.01035, size = 306, normalized size = 1.83

$$\frac{1}{5}aex^5 + \frac{1}{3}adx^3 + \frac{1}{12}\left(4x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd + \frac{1}{80}\left(16x^5\operatorname{arcsch}(cx) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + l
og(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsch(c*x) - (2*
(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1
)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4
 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

Fricas [A] time = 3.17026, size = 617, normalized size = 3.69

$$24ac^5ex^5 + 40ac^5dx^3 + 8\left(5bc^5d + 3bc^5e\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + \left(20bc^2d - 9be\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - 8\left(5bc^5d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*
sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (20*b*c^2*d - 9*b*e)*log(c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 - 5*
b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (
6*b*c^4*e*x^4 + (20*b*c^4*d - 9*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
)/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b \operatorname{acsch}(cx)\right) \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)\left(b \operatorname{arcsch}(cx) + a\right)x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

3.78 $\int \left(d+ex^2\right)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=115

$$dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(6c^2d-e\right)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{6c^2\sqrt{-c^2x^2}}+\frac{bex^2\sqrt{-c^2x^2-1}}{6c\sqrt{-c^2x^2}}$$

```
[Out] (b*e*x^2*Sqrt[-1 - c^2*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + d*x*(a + b*ArcCsch[c*
x]) + (e*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(6*c^2*d - e)*x*ArcTan[(c*x)/Sqrt
[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.0556023, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6292, 12, 388, 217, 203}

$$dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(6c^2d-e\right)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{6c^2\sqrt{-c^2x^2}}+\frac{bex^2\sqrt{-c^2x^2-1}}{6c\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*x^2*Sqrt[-1 - c^2*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + d*x*(a + b*ArcCsch[c*
x]) + (e*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(6*c^2*d - e)*x*ArcTan[(c*x)/Sqrt
[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rule 6292

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{3d+ex^2}{3\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{3d+ex^2}{\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}}$$

$$= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{\left(bc\left(6d-\frac{e}{c^2}\right)\right.}{6\sqrt{}}$$

$$= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{\left(bc\left(6d-\frac{e}{c^2}\right)\right.}{}$$

$$= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\left(6d-\frac{e}{c^2}\right)x}{6\sqrt{}}$$

Mathematica [A] time = 0.262972, size = 135, normalized size = 1.17

$$adx+\frac{1}{3}aex^3+\frac{bdx\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+\frac{bex^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{6c}-\frac{be\log\left(x\left(\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1\right)\right)}{6c^3}+bdx\text{csch}^{-1}(cx)+\frac{1}{3}bex^3\text{csch}^{-1}(c$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*x
*ArcCsch[c*x] + (b*e*x^3*ArcCsch[c*x])/3 + (b*d*Sqrt[1 + 1/(c^2*x^2)]*x*Arc
Sinh[c*x])/Sqrt[1 + c^2*x^2] - (b*e*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)
])])/(6*c^3)
```

Maple [A] time = 0.209, size = 126, normalized size = 1.1

$$\frac{1}{c}\left(\frac{a}{c^2}\left(\frac{ec^3x^3}{3}+xc^3d\right)+\frac{b}{c^2}\left(\frac{\operatorname{arccsch}(cx)\,ec^3x^3}{3}+\operatorname{arccsch}(cx)\,c^3dx+\frac{1}{6\,cx}\sqrt{c^2x^2+1}\left(6\,c^2d\operatorname{Arcsinh}(cx)+ecx\sqrt{c^2x^2+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c*(a/c^2*(1/3*e*c^3*x^3+x*c^3*d)+b/c^2*(1/3*arccsch(c*x)*e*c^3*x^3+arccsc
h(c*x)*c^3*d*x+1/6*(c^2*x^2+1)^(1/2)*(6*c^2*d*arcsinh(c*x)+e*c*x*(c^2*x^2+1
)^(1/2)-e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.02023, size = 200, normalized size = 1.74

$$\frac{1}{3}\,aex^3+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be+adx+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2
*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/
(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d/c
```

Fricas [B] time = 2.95878, size = 544, normalized size = 4.73

$$\frac{2\,ac^3ex^3 + bc^2ex^2\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 6\,ac^3dx + 2\left(3\,bc^3d + bc^3e\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - \left(6\,bc^2d - be\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - \cdots}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + b*c^2*e*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 6*a*c^3*d*
x + 2*(3*b*c^3*d + b*c^3*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - (6*b*c^2*d - b*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(3*b
*c^3*d + b*c^3*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c
^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{acsch}\,(cx))\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)(b\,\mathrm{arcsch}\,(cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

3.79 $\int \frac{(d+ex^2)(a+b\text{csch}^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=91

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}-\frac{bex\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}$$

```
[Out] (b*c*d*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] - (d*(a + b*ArcCsch[c*x]))/x +
e*x*(a + b*ArcCsch[c*x]) - (b*e*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(
c^2*x^2)]
```

Rubi [A] time = 0.0676791, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6302, 451, 217, 203}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}-\frac{bex\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (b*c*d*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] - (d*(a + b*ArcCsch[c*x]))/x +
e*x*(a + b*ArcCsch[c*x]) - (b*e*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(
c^2*x^2)]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
```

```
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int \frac{-d+ex^2}{x^2\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcex)\int \frac{1}{\sqrt{-1-c^2x^2}}\,\cdots}{\sqrt{-c^2x^2}}$$

$$=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcex)\operatorname{Subst}\left(\int \cdots\right)}{\sqrt{\cdots}}$$

$$=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{bex\tan^{-1}\left(\frac{cx}{\sqrt{-1-c^2\cdots}}\right)}{\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.12998, size = 89, normalized size = 0.98

$$-\frac{ad}{x}+aex+bcd\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}-\frac{bd\operatorname{csch}^{-1}(cx)}{x}+bex\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x + b*c*d*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsch[c*x
])/x + b*e*x*ArcCsch[c*x] + (b*e*Sqrt[1 + 1/(c^2*x^2)]*x*ArcSinh[c*x])/Sqrt
[1 + c^2*x^2]
```

Maple [A] time = 0.194, size = 107, normalized size = 1.2

$$c\left(\frac{a}{c^2}\left(cxe-\frac{cd}{x}\right)+\frac{b}{c^2}\left(\operatorname{arccsch}(cx)\,cxe-\frac{\operatorname{arccsch}(cx)\,cd}{x}+\frac{1}{c^2x^2}\sqrt{c^2x^2+1}\left(c^2d\sqrt{c^2x^2+1}+e\operatorname{Arcsinh}(cx)\,cx\right)\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x)
```

```
[Out] c*(a/c^2*(c*x*e-c*d/x)+b/c^2*(arccsch(c*x)*c*x*e-arccsch(c*x)*c*d/x+(c^2*x^
2+1)^(1/2)*(c^2*d*(c^2*x^2+1)^(1/2)+e*arcsinh(c*x)*c*x)/c^2/x^2/((c^2*x^2+1
)/c^2/x^2)^(1/2)))
```

Maxima [A] time = 0.974879, size = 113, normalized size = 1.24

$$\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)bd+aex+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)be}{2\,c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arccsch
(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*
e/c - a*d/x
```

Fricas [B] time = 2.76196, size = 493, normalized size = 5.42

$$\frac{bc^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + bc^2dx + acex^2 - bex\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - acd - (bcd - bce)x\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + (bcd - bce)x}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + b*c^2*d*x + a*c*e*x^2 - b*e*x*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - a*c*d - (b*c*d - b*c*e)*x*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c*d - b*c*e)*x*log(c*x*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*e*x^2 - b*c*d + (b*c*d - b*c*
e)*x)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acsch}(cx))\left(d + ex^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\operatorname{arcsch}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.80 $\int \frac{\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4} dx$

Optimal. Leaf size=109

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}-\frac{bc\sqrt{-c^2x^2-1}\left(2c^2d-9e\right)}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}$$

```
[Out] -(b*c*(2*c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/(9*Sqrt[-(c^2*x^2)]) + (b*c*d*Sqr
t[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(3*x^3
) - (e*(a + b*ArcCsch[c*x]))/x
```

Rubi [A] time = 0.079277, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6302, 12, 453, 264}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}-\frac{bc\sqrt{-c^2x^2-1}\left(2c^2d-9e\right)}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] -(b*c*(2*c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/(9*Sqrt[-(c^2*x^2)]) + (b*c*d*Sqr
t[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(3*x^3
) - (e*(a + b*ArcCsch[c*x]))/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
```

```
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}-\frac{(bcx)\int \frac{-d-3ex^2}{3x^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\\
&= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}-\frac{(bcx)\int \frac{-d-3ex^2}{x^4\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}}\\
&= \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}-\frac{\left(bc\left(2c^2d-9e\right)x\right)\ldots}{9\sqrt{-c^2\ldots}}\\
&= -\frac{bc\left(2c^2d-9e\right)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csc}\ldots\right.}{x}
\end{aligned}
$$

Mathematica [A] time = 0.0851028, size = 68, normalized size = 0.62

$$
\frac{-3a\left(d+3ex^2\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(-2c^2dx^2+d+9ex^2\right)-3b\text{csch}^{-1}(cx)\left(d+3ex^2\right)}{9x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 9*e*x^
2) - 3*b*(d + 3*e*x^2)*ArcCsch[c*x])/(9*x^3)
```

Maple [A] time = 0.2, size = 122, normalized size = 1.1

$$c^3\left(\frac{a}{c^2}\left(-\frac{e}{cx}-\frac{d}{3\,cx^3}\right)+\frac{b}{c^2}\left(-\frac{\operatorname{arccsch}(cx)\,e}{cx}-\frac{\operatorname{arccsch}(cx)\,d}{3\,cx^3}-\frac{\left(c^2x^2+1\right)\left(2\,c^4dx^2-9\,c^2x^2e-c^2d\right)}{9\,c^4x^4}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x)
```

```
[Out] c^3*(a/c^2*(-e/c/x-1/3/c*d/x^3)+b/c^2*(-arccsch(c*x)*e/c/x-1/3*arccsch(c*x)
/c*d/x^3-1/9*(c^2*x^2+1)*(2*c^4*d*x^2-9*c^2*e*x^2-c^2*d)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/c^4/x^4))
```

Maxima [A] time = 0.994452, size = 123, normalized size = 1.13

$$\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)be+\frac{1}{9}\,bd\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arcsch}(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2)
 + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - a*e/x
- 1/3*a*d/x^3
```

Fricas [A] time = 2.35906, size = 234, normalized size = 2.15

$$-\frac{9\,aex^2+3\,ad+3\left(3\,bex^2+bd\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(bcdx-\left(2\,bc^3d-9\,bce\right)x^3\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)) - (b*c*d*x - (2*b*c^3*d - 9*b*c*e)*x^3)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{acsch}\,(cx))\left(d+ex^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)(b\,\mathrm{arcsch}\,(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

3.81 $\int \frac{(d+ex^2)(a+b\text{csch}^{-1}(cx))}{x^6} dx$

Optimal. Leaf size=158

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}+\frac{2bc^3\sqrt{-c^2x^2-1}\left(12c^2d-25e\right)}{225\sqrt{-c^2x^2}}-\frac{bc\sqrt{-c^2x^2-1}\left(12c^2d-25e\right)}{225x^2\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}}$$

```
[Out] (2*b*c^3*(12*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c^2*x^2)]) + (b*
c*d*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (b*c*(12*c^2*d - 25*e)*
Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(
5*x^5) - (e*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rubi [A] time = 0.102568, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 6302, 12, 453, 271, 264}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}+\frac{2bc^3\sqrt{-c^2x^2-1}\left(12c^2d-25e\right)}{225\sqrt{-c^2x^2}}-\frac{bc\sqrt{-c^2x^2-1}\left(12c^2d-25e\right)}{225x^2\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```


```
[Out] (2*b*c^3*(12*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c^2*x^2)]) + (b*
c*d*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (b*c*(12*c^2*d - 25*e)*
Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(
5*x^5) - (e*(a + b*ArcCsch[c*x]))/(3*x^3)
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\operatorname{csch}^{-1}(cx))}{x^6}\,dx = -\frac{d(a+b\operatorname{csch}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{csch}^{-1}(cx))}{3x^3} - \frac{(bcx)\int \frac{-3d-5ex^2}{15x^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{d(a+b\operatorname{csch}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{csch}^{-1}(cx))}{3x^3} - \frac{(bcx)\int \frac{-3d-5ex^2}{x^6\sqrt{-1-c^2x^2}}\,dx}{15\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{d(a+b\operatorname{csch}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{csch}^{-1}(cx))}{3x^3} - \frac{(bc(12c^2d-25e)x)\ldots}{75\sqrt{-c\ldots}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d(a+b\operatorname{csch}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{csc}\ldots}{3x^3}$$

$$= \frac{2bc^3(12c^2d-25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \ldots$$

Mathematica [A] time = 0.121658, size = 93, normalized size = 0.59

$$\frac{-15a(3d+5ex^2)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(3d(8c^4x^4-4c^2x^2+3)+25ex^2(1-2c^2x^2)\right)-15b\operatorname{csch}^{-1}(cx)(3d+5ex^2)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(25*e*x^2*(1 - 2*c^2*x
^2) + 3*d*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcCsch[c*x])
/(225*x^5)
```

Maple [A] time = 0.197, size = 140, normalized size = 0.9

$$c^5\left(\frac{a}{c^2}\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)+\frac{b}{c^2}\left(-\frac{\operatorname{arccsch}(cx)\,d}{5c^3x^5}-\frac{\operatorname{arccsch}(cx)\,e}{3c^3x^3}+\frac{(c^2x^2+1)(24x^4c^6d-50c^4ex^4-12c^4dx^2+25c^2x^2\ldots}{225c^6x^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x)
```

```
[Out] c^5*(a/c^2*(-1/5/c^3*d/x^5-1/3*e/c^3/x^3)+b/c^2*(-1/5*arccsch(c*x)/c^3*d/x^
5-1/3*arccsch(c*x)*e/c^3/x^3+1/225*(c^2*x^2+1)*(24*c^6*d*x^4-50*c^4*e*x^4-1
2*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))
```

Maxima [A] time = 1.00057, size = 178, normalized size = 1.13

$$\frac{1}{75}\,bd\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\mathrm{arcsch}\,(cx)}{x^5}\right)+\frac{1}{9}\,be\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*d*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) +
 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) + 1/9*b*e*((c^4*(1/
(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3)
 - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Fricas [A] time = 2.60352, size = 294, normalized size = 1.86

$$-\frac{75\,aex^2+45\,ad+15\left(5\,bex^2+3\,bd\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(2\left(12\,bc^5d-25\,bc^3e\right)x^5+9\,bcdx-\left(12\,bc^3d-25\,bce\right)x^3\right)\sqrt{\frac{c^2x}{c^2}}}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) - (2*(12*b*c^5*d - 25*b*c^3*e)*x^5 + 9*b*c*d*x
 - (12*b*c^3*d - 25*b*c*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{acsch}\,(cx))\left(d+ex^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b \operatorname{arcsch}(cx) + a)}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

3.82 $\int \frac{(d+ex^2)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=205

$$-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{8bc^5\sqrt{-c^2x^2-1}\left(30c^2d-49e\right)}{3675\sqrt{-c^2x^2}}+\frac{4bc^3\sqrt{-c^2x^2-1}\left(30c^2d-49e\right)}{3675x^2\sqrt{-c^2x^2}}-\frac{bc\sqrt{-}}{}$$

```
[Out] (-8*b*c^5*(30*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(3675*Sqrt[-(c^2*x^2)]) + (
b*c*d*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]) - (b*c*(30*c^2*d - 49*e
)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2)]) + (4*b*c^3*(30*c^2*d - 49
*e)*Sqrt[-1 - c^2*x^2])/(3675*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x
]))/(7*x^7) - (e*(a + b*ArcCsch[c*x]))/(5*x^5)
```

Rubi [A] time = 0.121917, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 6302, 12, 453, 271, 264}

$$-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{8bc^5\sqrt{-c^2x^2-1}\left(30c^2d-49e\right)}{3675\sqrt{-c^2x^2}}+\frac{4bc^3\sqrt{-c^2x^2-1}\left(30c^2d-49e\right)}{3675x^2\sqrt{-c^2x^2}}-\frac{bc\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-8*b*c^5*(30*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(3675*Sqrt[-(c^2*x^2)]) + (
b*c*d*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]) - (b*c*(30*c^2*d - 49*e
)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2)]) + (4*b*c^3*(30*c^2*d - 49
*e)*Sqrt[-1 - c^2*x^2])/(3675*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x
]))/(7*x^7) - (e*(a + b*ArcCsch[c*x]))/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\text{csch}^{-1}(cx))}{x^8}\,dx = -\frac{d(a+b\text{csch}^{-1}(cx))}{7x^7} - \frac{e(a+b\text{csch}^{-1}(cx))}{5x^5} - \frac{(bcx)\int \frac{-5d-7ex^2}{35x^8\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{d(a+b\text{csch}^{-1}(cx))}{7x^7} - \frac{e(a+b\text{csch}^{-1}(cx))}{5x^5} - \frac{(bcx)\int \frac{-5d-7ex^2}{x^8\sqrt{-1-c^2x^2}}\,dx}{35\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{d(a+b\text{csch}^{-1}(cx))}{7x^7} - \frac{e(a+b\text{csch}^{-1}(cx))}{5x^5} - \frac{(bc(30c^2d-49e)\,x}{245\sqrt{-}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} - \frac{d(a+b\text{csch}^{-1}(cx))}{7x^7} - \frac{e(a+b\text{cs}}{5}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{4bc^3(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675x^2\sqrt{-c^2x^2}} -$$

$$= -\frac{8bc^5(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.145752, size = 109, normalized size = 0.53

$$\frac{-105a(5d+7ex^2) + bcx\sqrt{\frac{1}{c^2x^2}+1}\left(49ex^2(8c^4x^4-4c^2x^2+3) - 15d(16c^6x^6-8c^4x^4+6c^2x^2-5)\right) - 105b\text{csch}^{-1}(cx)\,(5}{3675x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(49*e*x^2*(3 - 4*c^2*
x^2 + 8*c^4*x^4) - 15*d*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*
(5*d + 7*e*x^2)*ArcCsch[c*x])/(3675*x^7)
```

Maple [A] time = 0.198, size = 158, normalized size = 0.8

$$c^7\left(\frac{a}{c^2}\left(-\frac{d}{7c^5x^7} - \frac{e}{5c^5x^5}\right) + \frac{b}{c^2}\left(-\frac{\text{arccsch}(cx)\,d}{7c^5x^7} - \frac{\text{arccsch}(cx)\,e}{5c^5x^5} - \frac{(c^2x^2+1)(240c^8dx^6 - 392c^6x^6e - 120x^4c^6d + 19}{3675c^8x^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x)
```

```
[Out] c^7*(a/c^2*(-1/7/c^5*d/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arccsch(c*x)/c^5*d/x^
7-1/5*arccsch(c*x)*e/c^5/x^5-1/3675*(c^2*x^2+1)*(240*c^8*d*x^6-392*c^6*e*x^
6-120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2-147*c^2*e*x^2-75*c^2*d)/((c^2*x^
2+1)/c^2/x^2)^(1/2)/c^8/x^8))
```

Maxima [A] time = 1.01775, size = 223, normalized size = 1.09

$$\frac{1}{245}\,bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\mathrm{arcsch}\,(cx)}{x^7}\right)+\frac{1}{75}\,be\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2)
+ 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*arc
csch(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*
x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) -
1/5*a*e/x^5 - 1/7*a*d/x^7
```

Fricas [A] time = 2.70333, size = 350, normalized size = 1.71

$$-\frac{735\,aex^2+525\,ad+105\left(7\,bex^2+5\,bd\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(8\left(30\,bc^7d-49\,bc^5e\right)x^7-4\left(30\,bc^5d-49\,bc^3e\right)x^5-75\,bcdx\right.}{3675\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*(30*b*c^7*d - 49*b*c^5*e)*x^7 - 4*(30
*b*c^5*d - 49*b*c^3*e)*x^5 - 75*b*c*d*x + 3*(30*b*c^3*d - 49*b*c*e)*x^3)*sq
```

```
rt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acsch}\,(cx))\left(d + ex^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**8,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**8, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\,\text{arcsch}\,(cx) + a)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^8, x)
```

3.83 $\int x^5 \left(d+ex^2\right) \left(a+b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=204

$$\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\left(-c^2x^2-1\right)^{5/2}\left(4c^2d-9e\right)}{120c^7\sqrt{-c^2x^2}}+\frac{bx\left(-c^2x^2-1\right)^{3/2}\left(8c^2d-9e\right)}{72c^7\sqrt{-c^2x^2}}+\frac{bx\sqrt{}}{}$$

```
[Out] (b*(4*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2])/(24*c^7*Sqrt[-(c^2*x^2)]) + (b*(8*
c^2*d - 9*e)*x*(-1 - c^2*x^2)^(3/2))/(72*c^7*Sqrt[-(c^2*x^2)]) + (b*(4*c^2*
d - 9*e)*x*(-1 - c^2*x^2)^(5/2))/(120*c^7*Sqrt[-(c^2*x^2)]) - (b*e*x*(-1 -
c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*x^2)]) + (d*x^6*(a + b*ArcCsch[c*x]))/6
+ (e*x^8*(a + b*ArcCsch[c*x]))/8
```

Rubi [A] time = 0.1613, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6302, 12, 446, 77}

$$\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\left(-c^2x^2-1\right)^{5/2}\left(4c^2d-9e\right)}{120c^7\sqrt{-c^2x^2}}+\frac{bx\left(-c^2x^2-1\right)^{3/2}\left(8c^2d-9e\right)}{72c^7\sqrt{-c^2x^2}}+\frac{bx\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```


```
[Out] (b*(4*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2])/(24*c^7*Sqrt[-(c^2*x^2)]) + (b*(8*
c^2*d - 9*e)*x*(-1 - c^2*x^2)^(3/2))/(72*c^7*Sqrt[-(c^2*x^2)]) + (b*(4*c^2*
d - 9*e)*x*(-1 - c^2*x^2)^(5/2))/(120*c^7*Sqrt[-(c^2*x^2)]) - (b*e*x*(-1 -
c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*x^2)]) + (d*x^6*(a + b*ArcCsch[c*x]))/6
+ (e*x^8*(a + b*ArcCsch[c*x]))/8
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
```

```
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^5 \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5\left(4d+3ex^2\right)}{24\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5\left(4d+3ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{24\sqrt{-c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\,\text{Subst}\left(\int\frac{x^2(4d+3ex)}{\sqrt{-1-c^2x}}\,dx,\right.}{48\sqrt{-c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\,\text{Subst}\left(\int\left(\frac{4c^2d-3e}{c^6\sqrt{-1-c^2x}}+\right.\right.}{}$$

$$=\frac{b\left(4c^2d-3e\right)x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}+\frac{b\left(8c^2d-9e\right)x\left(-1-c^2x^2\right)^{3/2}}{72c^7\sqrt{-c^2x^2}}+\frac{b\left(4c^2d-9e\right)x\left(-1\right.}{120c^7\sqrt{-c^2}}$$

Mathematica [A] time = 0.248026, size = 114, normalized size = 0.56

$$\frac{x\left(105ax^5\left(4d+3ex^2\right)+\frac{b\sqrt{\frac{1}{c^2x^2}+1}\left(c^6\left(84dx^4+45ex^6\right)-2c^4\left(56dx^2+27ex^4\right)+8c^2\left(28d+9ex^2\right)-144e\right)}{c^7}+105bx^5\text{csch}^{-1}(cx)\left(4d+3ex^2\right)\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-144*e + 8*c^2*(2
8*d + 9*e*x^2) - 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/
c^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsch[c*x]))/2520
```

Maple [A] time = 0.2, size = 152, normalized size = 0.8

$$\frac{1}{c^6}\left(\frac{a}{c^2}\left(\frac{ec^8x^8}{8}+\frac{c^8x^6d}{6}\right)+\frac{b}{c^2}\left(\frac{\text{arccsch}\,(cx)\,ec^8x^8}{8}+\frac{\text{arccsch}\,(cx)\,c^8x^6d}{6}+\frac{\left(c^2x^2+1\right)\left(45\,c^6x^6e+84\,x^4c^6d-54\,c^4ex^4-11\right.}{2520\,cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^6*(a/c^2*(1/8*e*c^8*x^8+1/6*c^8*x^6*d)+b/c^2*(1/8*arccsch(c*x)*e*c^8*x^
8+1/6*arccsch(c*x)*c^8*x^6*d+1/2520*(c^2*x^2+1)*(45*c^6*e*x^6+84*c^6*d*x^4-
54*c^4*e*x^4-112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d-144*e)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/c/x))
```

Maxima [A] time = 1.00615, size = 238, normalized size = 1.17

$$\frac{1}{8}\,aex^8+\frac{1}{6}\,adx^6+\frac{1}{90}\left(15\,x^6\,\mathrm{arcsch}\left(cx\right)+\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)bd+\frac{1}{280}\left(35\,x^8\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*
x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2
) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1
)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)
^(3/2) - 35*x*sqrt(1/(c^2*x^2) + 1))/c^7)*b*e
```

Fricas [A] time = 3.0506, size = 378, normalized size = 1.85

$$\frac{315\,ac^7ex^8+420\,ac^7dx^6+105\left(3\,bc^7ex^8+4\,bc^7dx^6\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(45\,bc^6ex^7+6\left(14\,bc^6d-9\,bc^4e\right)x^5-8\left(14\,bc^4d\right.\right.}{2520\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d*
x^6)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (45*b*c^6*e*x^7 +
 6*(14*b*c^6*d - 9*b*c^4*e)*x^5 - 8*(14*b*c^4*d - 9*b*c^2*e)*x^3 + 16*(14*b
```

```
*c^2*d - 9*b*e)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{acsch}(cx)) \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)(b \operatorname{arcsch}(cx) + a)x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

3.84 $\int x^3\left(d+ex^2\right)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=159

$$\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(-c^2x^2-1\right)^{3/2}\left(3c^2d-4e\right)}{36c^5\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\left(3c^2d-2e\right)}{12c^5\sqrt{-c^2x^2}}+\frac{bex\left(-\right.}{30c}$$

```
[Out] -(b*(3*c^2*d - 2*e)*x*Sqrt[-1 - c^2*x^2])/(12*c^5*Sqrt[-(c^2*x^2)]) - (b*(3
*c^2*d - 4*e)*x*(-1 - c^2*x^2)^(3/2))/(36*c^5*Sqrt[-(c^2*x^2)]) + (b*e*x*(-
1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + (d*x^4*(a + b*ArcCsch[c*x])
)/4 + (e*x^6*(a + b*ArcCsch[c*x]))/6
```

Rubi [A] time = 0.13391, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6302, 12, 446, 77}

$$\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(-c^2x^2-1\right)^{3/2}\left(3c^2d-4e\right)}{36c^5\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\left(3c^2d-2e\right)}{12c^5\sqrt{-c^2x^2}}+\frac{bex\left(-\right.}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -(b*(3*c^2*d - 2*e)*x*Sqrt[-1 - c^2*x^2])/(12*c^5*Sqrt[-(c^2*x^2)]) - (b*(3
*c^2*d - 4*e)*x*(-1 - c^2*x^2)^(3/2))/(36*c^5*Sqrt[-(c^2*x^2)]) + (b*e*x*(-
1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + (d*x^4*(a + b*ArcCsch[c*x])
)/4 + (e*x^6*(a + b*ArcCsch[c*x]))/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
```

```
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)dx=\frac{1}{4}dx^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^3\left(3d+2ex^2\right)}{12\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^3\left(3d+2ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{12\sqrt{-c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\,\mathrm{Subst}\left(\int\frac{x(3d+2ex)}{\sqrt{-1-c^2x}}\,dx,x\right.}{24\sqrt{-c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\,\mathrm{Subst}\left(\int\left(\frac{-3c^2d+2e}{c^4\sqrt{-1-c^2x}}+\right.\right.}{}$$

$$=-\frac{b\left(3c^2d-2e\right)x\sqrt{-1-c^2x^2}}{12c^5\sqrt{-c^2x^2}}-\frac{b\left(3c^2d-4e\right)x\left(-1-c^2x^2\right)^{3/2}}{36c^5\sqrt{-c^2x^2}}+\frac{bex\left(-1-c^2x^2\right)^{5/2}}{30c^5\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.207983, size = 97, normalized size = 0.61

$$\frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{\frac{1}{c^2x^2}+1}\left(3c^4\left(5dx^2+2ex^4\right)-2c^2\left(15d+4ex^2\right)+16e\right)}{c^5}+15bx^3\operatorname{csch}^{-1}(cx)\left(3d+2ex^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(16*e - 2*c^2*(15*d
 + 4*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*Ar
cCsch[c*x]))/180
```

Maple [A] time = 0.197, size = 134, normalized size = 0.8

$$\frac{1}{c^4}\left(\frac{a}{c^2}\left(\frac{c^6x^6e}{6}+\frac{x^4c^6d}{4}\right)+\frac{b}{c^2}\left(\frac{\operatorname{arccsch}(cx)c^6x^6e}{6}+\frac{\operatorname{arccsch}(cx)c^6x^4d}{4}+\frac{\left(c^2x^2+1\right)\left(6c^4ex^4+15c^4dx^2-8c^2x^2e-30\right.}{180cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^4*(a/c^2*(1/6*c^6*x^6*e+1/4*x^4*c^6*d)+b/c^2*(1/6*arccsch(c*x)*c^6*x^6*
e+1/4*arccsch(c*x)*c^6*x^4*d+1/180*(c^2*x^2+1)*(6*c^4*e*x^4+15*c^4*d*x^2-8*
c^2*e*x^2-30*c^2*d+16*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.02328, size = 185, normalized size = 1.16

$$\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd+\frac{1}{90}\left(15x^6\operatorname{arcsch}(cx)+\frac{3c^4x^5\left(\frac{1}{c^2x^2}+1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2
) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsch(c
*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/
2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e
```

Fricas [A] time = 3.3434, size = 323, normalized size = 2.03

$$\frac{30\,ac^5ex^6 + 45\,ac^5dx^4 + 15\left(2\,bc^5ex^6 + 3\,bc^5dx^4\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + \left(6\,bc^4ex^5 + \left(15\,bc^4d - 8\,bc^2e\right)x^3 - 2\left(15\,bc^2d - 8\,be\right)\right.}{180\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4)
*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e*x^5 + (15*
b*c^4*d - 8*b*c^2*e)*x^3 - 2*(15*b*c^2*d - 8*b*e)*x)*sqrt((c^2*x^2 + 1)/(c^
2*x^2)))/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + b\,\mathrm{acsch}\,(cx)\right)\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)\left(b\,\mathrm{arcsch}\,(cx) + a\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

3.85 $\int x\left(d+ex^2\right)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=146

$$\frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e}-\frac{bcd^2x\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{4e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\left(2c^2d-e\right)}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-c^2x^2-1\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}$$

```
[Out] (b*(2*c^2*d - e)*x*Sqrt[-1 - c^2*x^2])/(4*c^3*Sqrt[-(c^2*x^2)]) - (b*e*x*(-
1 - c^2*x^2)^(3/2))/(12*c^3*Sqrt[-(c^2*x^2)]) + ((d + e*x^2)^2*(a + b*ArcCs
ch[c*x]))/(4*e) - (b*c*d^2*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(4*e*Sqrt[-(c^2*x^
2)])
```

Rubi [A] time = 0.105749, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6300, 446, 88, 63, 205}

$$\frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e}-\frac{bcd^2x\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{4e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\left(2c^2d-e\right)}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-c^2x^2-1\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(2*c^2*d - e)*x*Sqrt[-1 - c^2*x^2])/(4*c^3*Sqrt[-(c^2*x^2)]) - (b*e*x*(-
1 - c^2*x^2)^(3/2))/(12*c^3*Sqrt[-(c^2*x^2)]) + ((d + e*x^2)^2*(a + b*ArcCs
ch[c*x]))/(4*e) - (b*c*d^2*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(4*e*Sqrt[-(c^2*x^
2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{(bcx)\int\frac{\left(d+ex^2\right)^2}{x\sqrt{-1-c^2x^2}}\,dx}{4e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{(bcx)\,\text{Subst}\left(\int\frac{(d+ex)^2}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{8e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{(bcx)\,\text{Subst}\left(\int\left(-\frac{e\left(-2c^2d+e\right)}{c^2\sqrt{-1-c^2x}}+\frac{d^2}{x\sqrt{-1-c^2x}}-\frac{c^2\sqrt{-1-c^2x}}{c^2}\right.\right.}{8e\sqrt{-c^2x^2}}$$

$$=\frac{b\left(2c^2d-e\right)x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-$$

$$=\frac{b\left(2c^2d-e\right)x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+$$

$$=\frac{b\left(2c^2d-e\right)x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-$$

Mathematica [A] time = 0.0953076, size = 77, normalized size = 0.53

$$\frac{x\left(3ac^3x\left(2d+ex^2\right)+b\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(6d+ex^2\right)-2e\right)+3bc^3x\text{csch}^{-1}(cx)\left(2d+ex^2\right)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-2*e + c^2*(6*d + e*
x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsch[c*x]))/(12*c^3)
```

Maple [A] time = 0.184, size = 115, normalized size = 0.8

$$\frac{1}{c^2}\left(\frac{a}{c^2}\left(\frac{c^4x^4e}{4}+\frac{x^2c^4d}{2}\right)+\frac{b}{c^2}\left(\frac{\text{arccsch}\,(cx)\,c^4x^4e}{4}+\frac{\text{arccsch}\,(cx)\,c^4x^2d}{2}+\frac{\left(c^2x^2+1\right)\left(c^2x^2e+6\,c^2d-2\,e\right)}{12\,cx}\frac{1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^2*(a/c^2*(1/4*c^4*x^4*e+1/2*x^2*c^4*d)+b/c^2*(1/4*arccsch(c*x)*c^4*x^4*
e+1/2*arccsch(c*x)*c^4*x^2*d+1/12*(c^2*x^2+1)*(c^2*e*x^2+6*c^2*d-2*e)/((c^2
*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.00803, size = 128, normalized size = 0.88

$$\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\,\mathrm{arcsch}\,(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd+\frac{1}{12}\left(3\,x^4\,\mathrm{arcsch}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x
*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e
```

Fricas [A] time = 2.7772, size = 266, normalized size = 1.82

$$\frac{3\,ac^3ex^4+6\,ac^3dx^2+3\left(bc^3ex^4+2\,bc^3dx^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(bc^2ex^3+2\left(3\,bc^2d-be\right)x\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e*x^3 + 2*(3*b*c^2*d
 - b*e)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{acsch}(cx)\right)\left(d+ex^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex^{2}+d\right)(b\operatorname{arcsch}(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

3.86 $\int \frac{(d+ex^2)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}bd\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)-d\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}+\frac{1}{2}bd\text{csch}^{-1}(c$$

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (b*d*ArcCsch[c*x]^2)/2 + (e*x^2*(a +
b*ArcCsch[c*x]))/2 - b*d*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + b*d*Arc
Csch[c*x]*Log[x^(-1)] - d*(a + b*ArcCsch[c*x])*Log[x^(-1)] - (b*d*PolyLog[2
, E^(2*ArcCsch[c*x])])/2
```

Rubi [A] time = 0.290918, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {6304, 14, 5789, 6742, 264, 2325, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}bd\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)-d\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}+\frac{1}{2}bd\text{csch}^{-1}(c$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (b*d*ArcCsch[c*x]^2)/2 + (e*x^2*(a +
b*ArcCsch[c*x]))/2 - b*d*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + b*d*Arc
Csch[c*x]*Log[x^(-1)] - d*(a + b*ArcCsch[c*x])*Log[x^(-1)] - (b*d*PolyLog[2
, E^(2*ArcCsch[c*x])])/2
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{-\frac{e}{2x^2}+d\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,\right.}{c}$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int\left(-\frac{e}{2x^2\sqrt{1+\frac{x^2}{c^2}}} + \right.\right.}{c}$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\,\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x,\right.}{c}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\text{csch}^{-1}(c\right.$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\text{csch}^{-1}(c\right.$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right)$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-1}(cx)\log\left(1-\right.$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-1}(cx)\log\left(1-\right.$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-1}(cx)\log\left(1-\right.$$

Mathematica [A] time = 0.134219, size = 93, normalized size = 0.81

$$\frac{bcd\text{PolyLog}\left(2, e^{-2\text{csch}^{-1}(cx)}\right) + 2acd\log(x) + acex^2 + bex\sqrt{\frac{1}{c^2x^2}+1} + bc\text{csch}^{-1}(cx)\left(ex^2 - 2d\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right) - bc}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x + a*c*e*x^2 - b*c*d*ArcCsch[c*x]^2 + b*c*ArcCs
ch[c*x]*(e*x^2 - 2*d*Log[1 - E^(-2*ArcCsch[c*x])]) + 2*a*c*d*Log[x] + b*c*d
*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*c)
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(a + b\,\mathrm{arccsch}(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,bc^2d\int \frac{x\log(x)}{2\left(\sqrt{c^2x^2+1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2+1} + 1\right)}\,dx - \frac{1}{2}\,bex^2\log(c) - \frac{1}{2}\,bex^2\log(x) + \frac{1}{2}\,aex^2 - bd\log(c)\log(x) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] 2*b*c^2*d*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqr
t(c^2*x^2 + 1) + 1), x) - 1/2*b*e*x^2*log(c) - 1/2*b*e*x^2*log(x) + 1/2*a*e
*x^2 - b*d*log(c)*log(x) - 1/2*b*d*log(x)^2 - 1/4*(2*log(c^2*x^2 + 1)*log(x
) + dilog(-c^2*x^2))*b*d + a*d*log(x) + 1/2*(b*e*x^2 + 2*b*d*log(x))*log(sq
rt(c^2*x^2 + 1) + 1) + 1/4*b*e*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2
 + 1/4*b*e*log(c^2*x^2 + 1)/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{aex^2 + ad + (bex^2 + bd)\,\mathrm{arcsch}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b \operatorname{arcsch}(cx) + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

3.87 $\int \frac{(d+ex^2)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{1}{2}be\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)-\frac{d\left(a+b\mathrm{csch}^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)+\frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{4x}-\frac{1}{4}bc^2d\mathrm{csch}^{-1}(cx)$$

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*d*ArcCsch[c*x])/4 + (b*e*ArcCs
ch[c*x]^2)/2 - (d*(a + b*ArcCsch[c*x]))/(2*x^2) - b*e*ArcCsch[c*x]*Log[1 -
E^(2*ArcCsch[c*x])] + b*e*ArcCsch[c*x]*Log[x^(-1)] - e*(a + b*ArcCsch[c*x])
*Log[x^(-1)] - (b*e*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rubi [A] time = 0.299569, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {6304, 14, 5789, 12, 6742, 321, 215, 2325, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}be\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)-\frac{d\left(a+b\mathrm{csch}^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)+\frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{4x}-\frac{1}{4}bc^2d\mathrm{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```


```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*d*ArcCsch[c*x])/4 + (b*e*ArcCs
ch[c*x]^2)/2 - (d*(a + b*ArcCsch[c*x]))/(2*x^2) - b*e*ArcCsch[c*x]*Log[1 -
E^(2*ArcCsch[c*x])] + b*e*ArcCsch[c*x]*Log[x^(-1)] - e*(a + b*ArcCsch[c*x])
*Log[x^(-1)] - (b*e*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```


Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = -\operatorname{Subst}\left(\int \frac{\left(e+dx^2\right)\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\operatorname{Subst}\left(\int \frac{dx^2+2e\log(x)}{2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x,\right.}{c}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\operatorname{Subst}\left(\int \frac{dx^2+2e\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x,\right.}{2c}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\operatorname{Subst}\left(\int\left(\frac{dx^2}{\sqrt{1+\frac{x^2}{c^2}}} + \frac{2e\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\right.\right.}{2c}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right.}{2c}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\mathrm{l}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - e$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + be\operatorname{csc}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - be\operatorname{csc}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - be\operatorname{csc}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - be\operatorname{csc}$$

Mathematica [A] time = 0.558171, size = 138, normalized size = 1.08

$$\frac{1}{4}\left(2be\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)-\frac{2ad}{x^2}+4ae\log(x)-\frac{bd\left(-c^2x^2+c^2x^2\sqrt{c^2x^2+1}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)-1\right)}{cx^3\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2bd\text{csch}^{-}}{x^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d)/x^2 - (2*b*d*ArcCsch[c*x])/x^2 - (b*d*(-1 - c^2*x^2 + c^2*x^2*Sqr
t[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]]))/(c*Sqrt[1 + 1/(c^2*x^2)]*x^3) -
 2*b*e*ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])]) + 4*a*e
*Log[x] + 2*b*e*PolyLog[2, E^(-2*ArcCsch[c*x])])/4
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)\left(a+b\text{arccsch}\left(cx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(4\,c^2\int \frac{x^2\log\left(x\right)}{c^2x^3+x}\,dx-2\,c^2\int \frac{x\log\left(x\right)}{c^2x^2+\left(c^2x^2+1\right)^{\frac{3}{2}}+1}\,dx-\left(\log\left(c^2x^2+1\right)-2\,\log\left(x\right)\right)\log\left(c\right)+\log\left(c^2x^2+1\right)\log\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log(
x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x))*
log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) + 2*
integrate(log(x)/(c^2*x^3 + x), x))*b*e + 1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2
) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
+ 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2) + a*
e*log(x) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{aex^2 + ad + \left(bex^2 + bd\right)\text{arcsch}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acsch}\,(cx))\left(d + ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)(b\,\text{arcsch}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

3.88 $\int x^2\left(d+ex^2\right)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=260

$$\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{-c^2x^2-1}\left(280c^4d^2-252c^2de+7\right.}{1680c^5\sqrt{-c^2x^2}}$$

```
[Out] (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 - c^2*x^2])/(1680*c^5*S
qrt[-(c^2*x^2)]) + (b*(84*c^2*d - 25*e)*e*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*
Sqrt[-(c^2*x^2)]) + (b*e^2*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)])
+ (d^2*x^3*(a + b*ArcCsch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsch[c*x]))/5 + (
e^2*x^7*(a + b*ArcCsch[c*x]))/7 + (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x
*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(1680*c^6*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.25813, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {270, 6302, 12, 1267, 459, 321, 217, 203}

$$\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{-c^2x^2-1}\left(280c^4d^2-252c^2de+7\right.}{1680c^5\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```


```
[Out] (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 - c^2*x^2])/(1680*c^5*S
qrt[-(c^2*x^2)]) + (b*(84*c^2*d - 25*e)*e*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*
Sqrt[-(c^2*x^2)]) + (b*e^2*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)])
+ (d^2*x^3*(a + b*ArcCsch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsch[c*x]))/5 + (
e^2*x^7*(a + b*ArcCsch[c*x]))/7 + (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x
*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(1680*c^6*Sqrt[-(c^2*x^2)])
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$=\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$=\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x$$

$$=\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+$$

$$=\frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+$$

$$=\frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+$$

$$=\frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+$$

Mathematica [A] time = 0.352764, size = 182, normalized size = 0.7

$$\frac{c^2x^2\left(16ac^5x\left(35d^2+42dex^2+15e^2x^4\right)+b\sqrt{\frac{1}{c^2x^2}+1}\left(8c^4\left(35d^2+21dex^2+5e^2x^4\right)-2c^2e\left(126d+25ex^2\right)+75e^2\right)\right)+b\left(-\right.}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 + 1/(c^2
*x^2)]*(75*e^2 - 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
```

```
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x]
 + b*(-280*c^4*d^2 + 252*c^2*d*e - 75*e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*
x])/(1680*c^7)
```

Maple [A] time = 0.19, size = 286, normalized size = 1.1

$$\frac{1}{c^3}\left(\frac{a}{c^4}\left(\frac{e^2c^7x^7}{7}+\frac{2\,c^7dex^5}{5}+\frac{x^3c^7d^2}{3}\right)+\frac{b}{c^4}\left(\frac{\operatorname{arccsch}(cx)\,e^2c^7x^7}{7}+\frac{2\operatorname{arccsch}(cx)\,c^7dex^5}{5}+\frac{\operatorname{arccsch}(cx)\,c^7x^3d^2}{3}-\frac{1}{1680}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^3*(a/c^4*(1/7*e^2*c^7*x^7+2/5*c^7*d*e*x^5+1/3*x^3*c^7*d^2)+b/c^4*(1/7*a
rccsch(c*x)*e^2*c^7*x^7+2/5*arccsch(c*x)*c^7*d*e*x^5+1/3*arccsch(c*x)*c^7*x
^3*d^2-1/1680*(c^2*x^2+1)^(1/2)*(-40*e^2*c^5*x^5*(c^2*x^2+1)^(1/2)-168*c^5*
d*e*x^3*(c^2*x^2+1)^(1/2)-280*d^2*c^5*x*(c^2*x^2+1)^(1/2)+280*d^2*c^4*arcsi
nh(c*x)+50*e^2*c^3*x^3*(c^2*x^2+1)^(1/2)+252*c^3*d*e*x*(c^2*x^2+1)^(1/2)-25
2*c^2*d*e*arcsinh(c*x)-75*e^2*c*x*(c^2*x^2+1)^(1/2)+75*e^2*arcsinh(c*x))/((
c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.00898, size = 535, normalized size = 2.06

$$\frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2+\frac{1}{40}\left(16\,x^5\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsch(c*x) +
(2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^
2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(16*
```

```
x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))
/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(
c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*e + 1
/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2
) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(
1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(sqrt(1/(c^2*x^
2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

Fricas [A] time = 4.20877, size = 891, normalized size = 3.43

$$\frac{240\,ac^7e^2x^7 + 672\,ac^7dex^5 + 560\,ac^7d^2x^3 + 16\left(35\,bc^7d^2 + 42\,bc^7de + 15\,bc^7e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + \left(280\,bc^4d^2 - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(35*
b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) - c*x + 1) + (280*b*c^4*d^2 - 252*b*c^2*d*e + 75*b*e^2)*log(c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) - c*x) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*
e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 16*(15*b*c^7*e^2*x^
7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e - 15*
b*c^7*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (40*b*c^6*e
^2*x^6 + 2*(84*b*c^6*d*e - 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 - 252*b*c^4*d
*e + 75*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a + b\operatorname{acsch}(cx)\right)\left(d + ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 (b \operatorname{arcsch}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^2, x)
```

3.89 $\int \left(d+ex^2\right)^2 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=197

$$d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(120c^4d^2-40c^2de+9e^2\right)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2}}\right.}{120c^4\sqrt{-c^2x^2}}$$

```
[Out] (b*(40*c^2*d - 9*e)*e*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) +
(b*e^2*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + d^2*x*(a + b*ArcCs
ch[c*x]) + (2*d*e*x^3*(a + b*ArcCsch[c*x]))/3 + (e^2*x^5*(a + b*ArcCsch[c*x
]))/5 - (b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*x*ArcTan[(c*x)/Sqrt[-1 - c^2*
x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.128402, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {194, 6292, 12, 1159, 388, 217, 203}

$$d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(120c^4d^2-40c^2de+9e^2\right)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2}}\right.}{120c^4\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(40*c^2*d - 9*e)*e*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) +
(b*e^2*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + d^2*x*(a + b*ArcCs
ch[c*x]) + (2*d*e*x^3*(a + b*ArcCsch[c*x]))/3 + (e^2*x^5*(a + b*ArcCsch[c*x
]))/5 - (b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*x*ArcTan[(c*x)/Sqrt[-1 - c^2*
x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6292

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
```

```
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bc\cdots}{\cdots}$$

$$= d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bc\cdots}{\cdots}$$

$$= \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+\cdots\right.$$

$$= \frac{b\left(40c^2d-9e\right)ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^{\cdots}$$

$$= \frac{b\left(40c^2d-9e\right)ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^{\cdots}$$

$$= \frac{b\left(40c^2d-9e\right)ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^{\cdots}$$

Mathematica [A] time = 0.22926, size = 149, normalized size = 0.76

$$\frac{c^2x\left(8ac^3\left(15d^2+10dex^2+3e^2x^4\right)+bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(40d+6ex^2\right)-9e\right)\right)+b\left(120c^4d^2-40c^2de+9e^2\right)\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+1}\cdots\right.\right.}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/(c^2*x^2
)]*x*(-9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^
2*x^4)*ArcCsch[c*x] + b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1
+ 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] time = 0.184, size = 217, normalized size = 1.1

$$\frac{1}{c}\left(\frac{a}{c^4}\left(\frac{e^2c^5x^5}{5}+\frac{2\,c^5dex^3}{3}+xc^5d^2\right)+\frac{b}{c^4}\left(\frac{\text{arccsch}\,(cx)\,e^2c^5x^5}{5}+\frac{2\,\text{arccsch}\,(cx)\,c^5x^3de}{3}+\text{arccsch}\,(cx)\,c^5xd^2+\frac{1}{120\,cx}\sqrt{c^2\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arccsch(c
*x)*e^2*c^5*x^5+2/3*arccsch(c*x)*c^5*x^3*d*e+arccsch(c*x)*c^5*x*d^2+1/120*(
c^2*x^2+1)^(1/2)*(120*d^2*c^4*arcsinh(c*x)+6*e^2*c^3*x^3*(c^2*x^2+1)^(1/2)+
40*c^3*d*e*x*(c^2*x^2+1)^(1/2)-40*c^2*d*e*arcsinh(c*x)-9*e^2*c*x*(c^2*x^2+1
)^(1/2)+9*e^2*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.0372, size = 387, normalized size = 1.96

$$\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{1}{6}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde+\frac{1}{80}\left(16x^5\operatorname{arcsch}(cx)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^
2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2
+ log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsch(c*x)
- (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2
) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1
)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/2*(2*c
*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1
) - 1))*b*d^2/c
```

Fricas [B] time = 3.81338, size = 792, normalized size = 4.02

$$\frac{24ac^5e^2x^5+80ac^5dex^3+120ac^5d^2x+8\left(15bc^5d^2+10bc^5de+3bc^5e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-\left(120bc^4d^2-40bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(15*b*c^5*
d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x + 1) - (120*b*c^4*d^2 - 40*b*c^2*d*e + 9*b*e^2)*log(c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - c*x) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*
d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e^2*x^4 + (40*b*c^
4*d*e - 9*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^2 (b \operatorname{arcsch}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a), x)
```

3.90 $\int \frac{(d+ex^2)^2(a+b\textbf{csch}^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=170

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}+2dex\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd^2\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}-\frac{bex\left(12c^2d-e\right)\tan^{-1}}{6c^2\sqrt{-c^2x^2}}$$

```
[Out] (b*c*d^2*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] + (b*e^2*x^2*Sqrt[-1 - c^2*x^
2])/(6*c*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/x + 2*d*e*x*(a + b*
ArcCsch[c*x]) + (e^2*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(12*c^2*d - e)*e*x*Ar
cTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.138769, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 6302, 12, 1265, 388, 217, 203}

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}+2dex\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd^2\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}-\frac{bex\left(12c^2d-e\right)\tan^{-1}}{6c^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] + (b*e^2*x^2*Sqrt[-1 - c^2*x^
2])/(6*c*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/x + 2*d*e*x*(a + b*
ArcCsch[c*x]) + (e^2*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(12*c^2*d - e)*e*x*Ar
cTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
```

```
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{(bc}{}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{(bc}{}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\right.$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}\right.$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}\right.$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}\right.$$

Mathematica [A] time = 0.214202, size = 134, normalized size = 0.79

$$\frac{c^2\left(2ac\left(-3d^2+6dex^2+e^2x^4\right)+bx\sqrt{\frac{1}{c^2x^2}+1}\left(6c^2d^2+e^2x^2\right)\right)+2bc^3\text{csch}^{-1}(cx)\left(-3d^2+6dex^2+e^2x^4\right)+bex\left(12c^2d-e\right.}{6c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (c^2*(b*Sqrt[1 + 1/(c^2*x^2)]*x*(6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*d
*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsch[c*x] +
b*(12*c^2*d - e)*e*x*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3*x)
```

Maple [A] time = 0.187, size = 189, normalized size = 1.1

$$c\left(\frac{a}{c^4}\left(\frac{c^3x^3e^2}{3}+2\,c^3xde-\frac{d^2c^3}{x}\right)+\frac{b}{c^4}\left(\frac{e^2\text{arccsch}\,(cx)\,c^3x^3}{3}+2\,\text{arccsch}\,(cx)\,c^3xde-\frac{\text{arccsch}\,(cx)\,d^2c^3}{x}+\frac{1}{6\,c^2x^2}\sqrt{c^2x^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x)
```

```
[Out] c*(a/c^4*(1/3*c^3*x^3*e^2+2*c^3*x*d*e-d^2*c^3/x)+b/c^4*(1/3*e^2*arccsch(c*x
)*c^3*x^3+2*arccsch(c*x)*c^3*x*d*e-arccsch(c*x)*d^2*c^3/x+1/6*(c^2*x^2+1)^(
1/2)*(6*d^2*c^4*(c^2*x^2+1)^(1/2)+12*c^3*d*e*arcsinh(c*x)*x+(c^2*x^2+1)^(1/
2)*c^2*x^2*e^2-arcsinh(c*x)*c*x*e^2)/c^2/x^2/((c^2*x^2+1)/c^2/x^2)^(1/2)))
```

Maxima [A] time = 1.03528, size = 258, normalized size = 1.52

$$\frac{1}{3}ae^2x^3 + \left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsch}(cx)}{x}\right)bd^2 + \frac{1}{12}\left(4x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d^2 + 1/12*(4*
x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) -
 log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c
)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1)
- log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d*e/c - a*d^2/x
```

Fricas [B] time = 3.26879, size = 747, normalized size = 4.39

$$2\,ac^3e^2x^4 + 6\,bc^4d^2x + 12\,ac^3dex^2 - 6\,ac^3d^2 - 2\left(3\,bc^3d^2 - 6\,bc^3de - bc^3e^2\right)x\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - \left(12\,bc^2de - be^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*(
3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
```

```
2)) - c*x + 1) - (12*b*c^2*d*e - b*e^2)*x*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) - c*x) + 2*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log(c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*
c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*d^2*x + b*c^2*e^2*x^3)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b \operatorname{arcsch}(cx) + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^2, x)
```

3.91 $\int \frac{(d+ex^2)^2 (a+b\,\mathbf{csch}^{-1}(cx))}{x^4} \, dx$

Optimal. Leaf size=164

$$-\frac{d^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de \left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+e^2x \left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bcd^2\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}-\frac{2bcd\sqrt{-c^2x^2-1}\left(c^2d-9e\right)}{9\sqrt{-c^2x^2}}$$

```
[Out] (-2*b*c*d*(c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/(9*Sqrt[-(c^2*x^2)]) + (b*c*d^2
*Sqrt[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/
(3*x^3) - (2*d*e*(a + b*ArcCsch[c*x]))/x + e^2*x*(a + b*ArcCsch[c*x]) - (b*
e^2*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(c^2*x^2)]
```

Rubi [A] time = 0.142695, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 6302, 12, 1265, 451, 217, 203}

$$-\frac{d^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de \left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+e^2x \left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bcd^2\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}-\frac{2bcd\sqrt{-c^2x^2-1}\left(c^2d-9e\right)}{9\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```


```
[Out] (-2*b*c*d*(c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/(9*Sqrt[-(c^2*x^2)]) + (b*c*d^2
*Sqrt[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/
(3*x^3) - (2*d*e*(a + b*ArcCsch[c*x]))/x + e^2*x*(a + b*ArcCsch[c*x]) - (b*
e^2*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(c^2*x^2)]
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
```

```
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^4} dx = -\frac{d^2 (a+b\text{csch}^{-1}(cx))}{3x^3} - \frac{2de (a+b\text{csch}^{-1}(cx))}{x} + e^2x (a+b\text{csch}^{-1}(cx)) - \frac{(bcx)\int}{}$$

$$= -\frac{d^2 (a+b\text{csch}^{-1}(cx))}{3x^3} - \frac{2de (a+b\text{csch}^{-1}(cx))}{x} + e^2x (a+b\text{csch}^{-1}(cx)) - \frac{(bcx)\int}{}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2 (a+b\text{csch}^{-1}(cx))}{3x^3} - \frac{2de (a+b\text{csch}^{-1}(cx))}{x} + e^2x (a+b\text{csch}$$

$$= -\frac{2bcd (c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2 (a+b\text{csch}^{-1}(cx))}{3x^3} - \frac{2de (a+}{}$$

$$= -\frac{2bcd (c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2 (a+b\text{csch}^{-1}(cx))}{3x^3} - \frac{2de (a+}{}$$

$$= -\frac{2bcd (c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2 (a+b\text{csch}^{-1}(cx))}{3x^3} - \frac{2de (a+}{}$$

Mathematica [A] time = 0.262792, size = 123, normalized size = 0.75

$$\frac{bcdx\sqrt{\frac{1}{c^2x^2}+1}\left(-2c^2dx^2+d+18ex^2\right)-3a\left(d^2+6dex^2-3e^2x^4\right)}{9x^3} + \frac{be^2\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)\right)}{c} - \frac{b\text{csch}^{-1}(cx)\left(d^2+6dex\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2 + 6*
d*e*x^2 - 3*e^2*x^4))/(9*x^3) - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsch[c*
x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/c
```

Maple [A] time = 0.191, size = 191, normalized size = 1.2

$$c^3\left(\frac{a}{c^4}\left(cxe^2-2\frac{cde}{x}-\frac{cd^2}{3x^3}\right)+\frac{b}{c^4}\left(\text{arccsch}(cx)\,cxe^2-2\frac{\text{arccsch}(cx)\,cde}{x}-\frac{\text{arccsch}(cx)\,d^2c}{3x^3}-\frac{1}{9c^4x^4}\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x)
```

```
[Out] c^3*(a/c^4*(c*x*e^2-2*c*d*e/x-1/3*d^2*c/x^3)+b/c^4*(arccsch(c*x)*c*x*e^2-2*
arccsch(c*x)*c*d*e/x-1/3*arccsch(c*x)*d^2*c/x^3-1/9*(c^2*x^2+1)^(1/2)*(2*(c
^2*x^2+1)^(1/2)*c^6*x^2*d^2-18*c^4*d*e*(c^2*x^2+1)^(1/2)*x^2-d^2*c^4*(c^2*x
^2+1)^(1/2)-9*e^2*arcsinh(c*x)*c^3*x^3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4
))
```

Maxima [A] time = 1.02882, size = 205, normalized size = 1.25

$$2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)bde+ae^2x+\frac{1}{9}bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arcsch}(cx)}{x^3}\right)+\frac{\left(2cx\operatorname{arcsch}(cx)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*((
c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*
x)/x^3) + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sq
rt(1/(c^2*x^2) + 1) - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Fricas [B] time = 3.01253, size = 743, normalized size = 4.53

$$\frac{9ace^2x^4-9be^2x^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-18acdex^2-3\left(bcd^2+6bcde-3bce^2\right)x^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)+3\left(bcd^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 - 9*b*e^2*x^3*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*
x) - 18*a*c*d*e*x^2 - 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x
^3*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*a*c*d^2 - 2*(b*c^4*
```

```
d^2 - 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2 + (b*c*
d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c*x)) + (b*c^2*d^2*x - 2*(b*c^4*d^2 - 9*b*c^2*d*e)*x^3)*sqrt((c^2*x^2 +
 1)/(c^2*x^2)))/(c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b \operatorname{arcsch}(cx) + a)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^4, x)
```

3.92 $\int \frac{\left(d+ex^2\right)^2\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=189

$$-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+\frac{bc\sqrt{-c^2x^2-1}\left(24c^4d^2-100c^2de+225e^2\right)}{225\sqrt{-c^2x^2}}+$$

```
[Out] (b*c*(24*c^4*d^2 - 100*c^2*d*e + 225*e^2)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c
^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (2*b*c
*d*(6*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(
a + b*ArcCsch[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsch[c*x]))/(3*x^3) - (e^2*
(a + b*ArcCsch[c*x]))/x
```

Rubi [A] time = 0.159792, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 6302, 12, 1265, 453, 264}

$$-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+\frac{bc\sqrt{-c^2x^2-1}\left(24c^4d^2-100c^2de+225e^2\right)}{225\sqrt{-c^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```


```
[Out] (b*c*(24*c^4*d^2 - 100*c^2*d*e + 225*e^2)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c
^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (2*b*c
*d*(6*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(
a + b*ArcCsch[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsch[c*x]))/(3*x^3) - (e^2*
(a + b*ArcCsch[c*x]))/x
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int}{}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int}{}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}\right.}{x}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd\left(6c^2d-25e\right)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(\right.}{}$$

$$= \frac{bc\left(24c^4d^2-100c^2de+225e^2\right)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd\left(6c^2d-25e\right.}{225x^2\sqrt{-}}$$

Mathematica [A] time = 0.216865, size = 126, normalized size = 0.67

$$\frac{-15a\left(3d^2+10dex^2+15e^2x^4\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(3d^2\left(8c^4x^4-4c^2x^2+3\right)-50dex^2\left(2c^2x^2-1\right)+225e^2x^4\right)-15b\text{csch}^{-1}(c}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(225
*e^2*x^4 - 50*d*e*x^2*(-1 + 2*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4))
 - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x])/(225*x^5)
```

Maple [A] time = 0.191, size = 191, normalized size = 1.

$$c^5\left(\frac{a}{c^4}\left(-\frac{e^2}{cx}-\frac{d^2}{5\,cx^5}-\frac{2\,de}{3\,cx^3}\right)+\frac{b}{c^4}\left(-\frac{\text{arccsch}\,(cx)\,e^2}{cx}-\frac{\text{arccsch}\,(cx)\,d^2}{5\,cx^5}-\frac{2\,\text{arccsch}\,(cx)\,de}{3\,cx^3}+\frac{\left(c^2x^2+1\right)\left(24\,c^8d^2x^4-1\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x)
```

```
[Out] c^5*(a/c^4*(-e^2/c/x-1/5*d^2/c/x^5-2/3/c*d*e/x^3)+b/c^4*(-arccsch(c*x)*e^2/
c/x-1/5*arccsch(c*x)*d^2/c/x^5-2/3*arccsch(c*x)/c*d*e/x^3+1/225*(c^2*x^2+1)
*(24*c^8*d^2*x^4-100*c^6*d*e*x^4-12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*
x^2+9*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))
```

Maxima [A] time = 1.0096, size = 236, normalized size = 1.25

$$\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)be^2+\frac{1}{75}bd^2\left(\frac{3c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsch}(cx)}{x^5}\right)+\frac{2}{9}bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(c
^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^
2) + 1))/c - 15*arccsch(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) + 1)^(3/2)
 - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - a*e^2/x - 2/3*a*d
*e/x^3 - 1/5*a*d^2/x^5
```

Fricas [A] time = 2.45351, size = 382, normalized size = 2.02

$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\left(15\,be^2x^4+10\,bdex^2+3\,bd^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(\left(24\,bc^5d^2-100\,bc^3de+225\,bce^2\right.\right.}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*
d*e*x^2 + 3*b*d^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - ((2
4*b*c^5*d^2 - 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x - 2*(6*b*c^3*d
^2 - 25*b*c*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^2}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b \operatorname{arcsch}(cx) + a)}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^6, x)
```

3.93 $\int \frac{(d+ex^2)^2 (a+b\,\textbf{csch}^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=249

$$-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{2bc^3\sqrt{-c^2x^2-1}\left(360c^4d^2-1176c^2de+1225e^2\right.}{11025\sqrt{-c^2x^2}}$$

```
[Out] (-2*b*c^3*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(1102
5*Sqrt[-(c^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]
) - (2*b*c*d*(15*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2
)]) + (b*c*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(110
25*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/(7*x^7) - (2*d*e*(a +
 b*ArcCsch[c*x]))/(5*x^5) - (e^2*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rubi [A] time = 0.196614, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 6302, 12, 1265, 453, 271, 264}

$$-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{2bc^3\sqrt{-c^2x^2-1}\left(360c^4d^2-1176c^2de+1225e^2\right.}{11025\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-2*b*c^3*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(1102
5*Sqrt[-(c^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]
) - (2*b*c*d*(15*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2
)]) + (b*c*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(110
25*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/(7*x^7) - (2*d*e*(a +
 b*ArcCsch[c*x]))/(5*x^5) - (e^2*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{}{}}{}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{}{}}{}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}\right.}{3x^3}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd\left(15c^2d-49e\right)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a\right.}{}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd\left(15c^2d-49e\right)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{bc\left(360c^4d^2-1176c^2de+122\right.}{11025x^2\sqrt{-c^2x}}$$

$$= -\frac{2bc^3\left(360c^4d^2-1176c^2de+1225e^2\right)\sqrt{-1-c^2x^2}}{11025\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd\left(15c^2\right.}{122}$$

Mathematica [A] time = 0.259464, size = 152, normalized size = 0.61

$$\frac{-105a\left(15d^2+42dex^2+35e^2x^4\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(-45d^2\left(16c^6x^6-8c^4x^4+6c^2x^2-5\right)+294dex^2\left(8c^4x^4-4c^2x^2+3\right)+1\right.}{11025x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1
225*e^2*x^4*(1 - 2*c^2*x^2) + 294*d*e*x^2*(3 - 4*c^2*x^2 + 8*c^4*x^4) - 45*
d^2*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2
 + 35*e^2*x^4)*ArcCsch[c*x])/(11025*x^7)
```

Maple [A] time = 0.207, size = 223, normalized size = 0.9

$$c^7\left(\frac{a}{c^4}\left(-\frac{d^2}{7\,c^3x^7}-\frac{2\,de}{5\,c^3x^5}-\frac{e^2}{3\,c^3x^3}\right)+\frac{b}{c^4}\left(-\frac{\text{arccsch}\,(cx)\,d^2}{7\,c^3x^7}-\frac{2\,\text{arccsch}\,(cx)\,de}{5\,c^3x^5}-\frac{\text{arccsch}\,(cx)\,e^2}{3\,c^3x^3}-\frac{\left(c^2x^2+1\right)\left(720\,c^{10}d^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x)
```

```
[Out] c^7*(a/c^4*(-1/7*d^2/c^3/x^7-2/5/c^3*d*e/x^5-1/3*e^2/c^3/x^3)+b/c^4*(-1/7*a
rccsch(c*x)*d^2/c^3/x^7-2/5*arccsch(c*x)/c^3*d*e/x^5-1/3*arccsch(c*x)*e^2/c
^3/x^3-1/11025*(c^2*x^2+1)*(720*c^10*d^2*x^6-2352*c^8*d*e*x^6-360*c^8*d^2*x
^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2-1225*c^4*e^2*x^4-882*c
^4*d*e*x^2-225*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^8/x^8))
```

Maxima [A] time = 1.04044, size = 313, normalized size = 1.26

$$\frac{1}{245}\,bd^2\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\mathrm{arcsch}\,(cx)}{x^7}\right)+\frac{2}{75}\,bde\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d^2*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2
) + 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*a
rccsch(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(
c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5
) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/
c - 3*arccsch(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

Fricas [A] time = 2.55013, size = 481, normalized size = 1.93

$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\left(35\,be^2x^4+42\,bdex^2+15\,bd^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(2\left(360\,bc^7d^2-1176\,bc^5\right.\right.}{11025\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + (2*(360*b*c^7*d^2 - 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 - (360*b*c^5*
d^2 - 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 - 225*b*c*d^2*x + 18*(15*b*c^3*d^2
 - 49*b*c*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^2}{x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**8,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**8, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b \operatorname{arcsch}(cx) + a)}{x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^8, x)
```

3.94 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=250

$$\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(-c^2x^2-1\right)^{3/2}\left(6c^4d^2-16c^2de+9\right.}{72c^7\sqrt{-c^2x^2}}$$

```
[Out] -(b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 - c^2*x^2])/(24*c^7*Sqrt[-(c^
2*x^2)]) - (b*(6*c^4*d^2 - 16*c^2*d*e + 9*e^2)*x*(-1 - c^2*x^2)^(3/2))/(72*
c^7*Sqrt[-(c^2*x^2)]) + (b*(8*c^2*d - 9*e)*e*x*(-1 - c^2*x^2)^(5/2))/(120*c
^7*Sqrt[-(c^2*x^2)]) - (b*e^2*x*(-1 - c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*x^
2)]) + (d^2*x^4*(a + b*ArcCsch[c*x]))/4 + (d*e*x^6*(a + b*ArcCsch[c*x]))/3
+ (e^2*x^8*(a + b*ArcCsch[c*x]))/8
```

Rubi [A] time = 0.234657, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 43, 6302, 12, 1251, 771}

$$\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\left(-c^2x^2-1\right)^{3/2}\left(6c^4d^2-16c^2de+9\right.}{72c^7\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```


```
[Out] -(b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 - c^2*x^2])/(24*c^7*Sqrt[-(c^
2*x^2)]) - (b*(6*c^4*d^2 - 16*c^2*d*e + 9*e^2)*x*(-1 - c^2*x^2)^(3/2))/(72*
c^7*Sqrt[-(c^2*x^2)]) + (b*(8*c^2*d - 9*e)*e*x*(-1 - c^2*x^2)^(5/2))/(120*c
^7*Sqrt[-(c^2*x^2)]) - (b*e^2*x*(-1 - c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*x^
2)]) + (d^2*x^4*(a + b*ArcCsch[c*x]))/4 + (d*e*x^6*(a + b*ArcCsch[c*x]))/3
+ (e^2*x^8*(a + b*ArcCsch[c*x]))/8
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)dx=\frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$=\frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$=\frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$=\frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$=-\frac{b\left(6c^4d^2-8c^2de+3e^2\right)x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}-\frac{b\left(6c^4d^2-16c^2de+9e^2\right)x\left(-1-c^2x^2\right)}{72c^7\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.402856, size = 159, normalized size = 0.64

$$\frac{x\left(105ax^3\left(6d^2+8dex^2+3e^2x^4\right)+\frac{b\sqrt{\frac{1}{c^2x^2}+1}\left(3c^6\left(70d^2x^2+56dex^4+15e^2x^6\right)-2c^4\left(210d^2+112dex^2+27e^2x^4\right)+8c^2e\left(56d+9ex^2\right)-144e^2\right)}{c^7}+105bx^3\right.}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-
144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) - 2*c^4*(210*d^2 + 112*d*e*x^2 + 27*e^2*
x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^
2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCsch[c*x]))/2520
```

Maple [A] time = 0.19, size = 214, normalized size = 0.9

$$\frac{1}{c^4}\left(\frac{a}{c^4}\left(\frac{e^2c^8x^8}{8}+\frac{c^8dex^6}{3}+\frac{x^4c^8d^2}{4}\right)+\frac{b}{c^4}\left(\frac{\operatorname{arccsch}(cx)e^2c^8x^8}{8}+\frac{\operatorname{arccsch}(cx)c^8dex^6}{3}+\frac{\operatorname{arccsch}(cx)c^8x^4d^2}{4}+\frac{(c^2x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^4*(a/c^4*(1/8*e^2*c^8*x^8+1/3*c^8*d*e*x^6+1/4*x^4*c^8*d^2)+b/c^4*(1/8*a
rccsch(c*x)*e^2*c^8*x^8+1/3*arccsch(c*x)*c^8*d*e*x^6+1/4*arccsch(c*x)*c^8*x
^4*d^2+1/2520*(c^2*x^2+1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2-5
4*c^4*e^2*x^4-224*c^4*d*e*x^2-420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e-144*e^
2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.05541, size = 329, normalized size = 1.32

$$\frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12}\left(3x^4\operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2 + \frac{1}{45}\left(15x^6\operatorname{arcsch}(cx) + \frac{3c^4\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsch(c*x) +
(c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d^2 +
1/45*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3
*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/280*(
35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1)^(7/2) - 21*c^4*x^5*(1/(c
^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 35*x*sqrt(1/(c^2*
x^2) + 1))/c^7)*b*e^2
```

Fricas [A] time = 2.78739, size = 506, normalized size = 2.02

$$\frac{315ac^7e^2x^8 + 840ac^7dex^6 + 630ac^7d^2x^4 + 105\left(3bc^7e^2x^8 + 8bc^7dex^6 + 6bc^7d^2x^4\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + \left(45bc^6e^2x^7 + 6\left(2\cdots\right.\right.}{2520c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3*
b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt((c^2*x^2 +
```

```
 1)/(c^2*x^2)) + 1)/(c*x)) + (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e - 9*b*c^4*
e^2)*x^5 + 2*(105*b*c^6*d^2 - 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 - 4*(105*b*
c^4*d^2 - 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^2 (b \operatorname{arcsch}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^3, x)
```

3.95 $\int x\left(d+ex^2\right)^2\left(a+b\textbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=203

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}+\frac{bx\sqrt{-c^2x^2-1}\left(3c^4d^2-3c^2de+e^2\right)}{6c^5\sqrt{-c^2x^2}}-\frac{bcd^3x\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{6e\sqrt{-c^2x^2}}-\frac{bex\left(-c^2x^2-1\right)^{3/2}\left(3c\right.}{18c^5\sqrt{-c^2x^2}}$$

```
[Out] (b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x*Sqrt[-1 - c^2*x^2])/(6*c^5*Sqrt[-(c^2*x^
2)]) - (b*(3*c^2*d - 2*e)*e*x*(-1 - c^2*x^2)^(3/2))/(18*c^5*Sqrt[-(c^2*x^2)
]) + (b*e^2*x*(-1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + ((d + e*x^2
)^3*(a + b*ArcCsch[c*x]))/(6*e) - (b*c*d^3*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(6
*e*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.155975, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6300, 446, 88, 63, 205}

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}+\frac{bx\sqrt{-c^2x^2-1}\left(3c^4d^2-3c^2de+e^2\right)}{6c^5\sqrt{-c^2x^2}}-\frac{bcd^3x\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{6e\sqrt{-c^2x^2}}-\frac{bex\left(-c^2x^2-1\right)^{3/2}\left(3c\right.}{18c^5\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x*Sqrt[-1 - c^2*x^2])/(6*c^5*Sqrt[-(c^2*x^
2)]) - (b*(3*c^2*d - 2*e)*e*x*(-1 - c^2*x^2)^(3/2))/(18*c^5*Sqrt[-(c^2*x^2)
]) + (b*e^2*x*(-1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + ((d + e*x^2
)^3*(a + b*ArcCsch[c*x]))/(6*e) - (b*c*d^3*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(6
*e*Sqrt[-(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\int\frac{\left(d+ex^2\right)^3}{x\sqrt{-1-c^2x^2}}dx}{6e\sqrt{-c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\,\text{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{-1-c^2x}}dx, x, x^2\right)}{12e\sqrt{-c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\,\text{Subst}\left(\int\left(\frac{e\left(3c^4d^2-3c^2de+e^2\right)}{c^4\sqrt{-1-c^2x}} + \frac{d^3}{x\sqrt{-1-c^2x}} - \frac{\left(3c^2 \right.}{}\right.\right.}{12e\sqrt{-c^2x^2}}$$

$$= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/2}}{18c^5\sqrt{-c^2x^2}} + \frac{be^2x\left(-1\right.}{30c^5\sqrt{}}$$

$$= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/2}}{18c^5\sqrt{-c^2x^2}} + \frac{be^2x\left(-1\right.}{30c^5\sqrt{}}$$

$$= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/2}}{18c^5\sqrt{-c^2x^2}} + \frac{be^2x\left(-1\right.}{30c^5\sqrt{}}$$

Mathematica [A] time = 0.302283, size = 123, normalized size = 0.61

$$\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right) + \frac{b\sqrt{\frac{1}{c^2x^2}+1}\left(3c^4\left(15d^2+5dex^2+e^2x^4\right)-2c^2e\left(15d+2ex^2\right)+8e^2\right)}{c^5} + 15bx\text{csch}^{-1}(cx)\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(8*e^2
- 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsch[c*x]))/90
```

Maple [A] time = 0.173, size = 182, normalized size = 0.9

$$\frac{1}{c^2}\left(\frac{a}{c^4}\left(\frac{e^2c^6x^6}{6}+\frac{c^6x^4de}{2}+\frac{x^2c^6d^2}{2}\right)+\frac{b}{c^4}\left(\frac{\text{arccsch}\,(cx)\,e^2c^6x^6}{6}+\frac{\text{arccsch}\,(cx)\,c^6x^4de}{2}+\frac{\text{arccsch}\,(cx)\,c^6x^2d^2}{2}+\frac{\left(c^2x^2+1\right)}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^2*(a/c^4*(1/6*e^2*c^6*x^6+1/2*c^6*x^4*d*e+1/2*x^2*c^6*d^2)+b/c^4*(1/6*a
rccsch(c*x)*e^2*c^6*x^6+1/2*arccsch(c*x)*c^6*x^4*d*e+1/2*arccsch(c*x)*c^6*x
^2*d^2+1/90*(c^2*x^2+1)*(3*c^4*e^2*x^4+15*c^4*d*e*x^2+45*c^4*d^2-4*c^2*e^2*
x^2-30*c^2*d*e+8*e^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A] time = 1.02566, size = 247, normalized size = 1.22

$$\frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2\,\mathrm{arcsch}\,(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2 + \frac{1}{6}\left(3\,x^4\,\mathrm{arcsch}\,(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,x\sqrt{\frac{1}{c^2x^2}\cdots}}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsch(c*x) + x*s
qrt(1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*
x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arcc
sch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1
)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e^2
```

Fricas [A] time = 2.72464, size = 410, normalized size = 2.02

$$\frac{15\,ac^5e^2x^6 + 45\,ac^5dex^4 + 45\,ac^5d^2x^2 + 15\left(bc^5e^2x^6 + 3\,bc^5dex^4 + 3\,bc^5d^2x^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + \left(3\,bc^4e^2x^5 + \left(15\,bc^4de\cdots\right.\right.}{90\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e^
2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2
```

```
*x^2)) + 1)/(c*x)) + (3*b*c^4*e^2*x^5 + (15*b*c^4*d*e - 4*b*c^2*e^2)*x^3 +
(45*b*c^4*d^2 - 30*b*c^2*d*e + 8*b*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c
^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,(a+b\,\mathrm{acsch}\,(cx))\left(d+ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2(b\,\mathrm{arcsch}\,(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x, x)
```

3.96 $\int \frac{(d+ex^2)^2(a+b\,\mathbf{csch}^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=178

$$-\frac{1}{2}bd^2\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+dex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\cdots$$

```
[Out] (b*(6*c^2*d - e)*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 + 1/(c^
2*x^2)]*x^3)/(12*c) + (b*d^2*ArcCsch[c*x]^2)/2 + d*e*x^2*(a + b*ArcCsch[c*x
]) + (e^2*x^4*(a + b*ArcCsch[c*x]))/4 - b*d^2*ArcCsch[c*x]*Log[1 - E^(2*Arc
Csch[c*x])] + b*d^2*ArcCsch[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsch[c*x])*Log
[x^(-1)] - (b*d^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rubi [A] time = 0.421449, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {6304, 266, 43, 5789, 6742, 453, 264, 2325, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}bd^2\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+dex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```



```
[Out] (b*(6*c^2*d - e)*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 + 1/(c^
2*x^2)]*x^3)/(12*c) + (b*d^2*ArcCsch[c*x]^2)/2 + d*e*x^2*(a + b*ArcCsch[c*x
]) + (e^2*x^4*(a + b*ArcCsch[c*x]))/4 - b*d^2*ArcCsch[c*x]*Log[1 - E^(2*Arc
Csch[c*x])] + b*d^2*ArcCsch[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsch[c*x])*Log
[x^(-1)] - (b*d^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```


Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
```

```
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)^2 \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= dex^2 (a+b\text{csch}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\text{csch}^{-1}(cx)) - d^2 (a+b\text{csch}^{-1}(cx)) \log\left(\frac{1}{x}\right)$$

$$= dex^2 (a+b\text{csch}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\text{csch}^{-1}(cx)) - d^2 (a+b\text{csch}^{-1}(cx)) \log\left(\frac{1}{x}\right)$$

$$= dex^2 (a+b\text{csch}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\text{csch}^{-1}(cx)) - d^2 (a+b\text{csch}^{-1}(cx)) \log\left(\frac{1}{x}\right)$$

$$= \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + dex^2 (a+b\text{csch}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\text{csch}^{-1}(cx)) + bd^2\text{csch}^{-1}($$

$$= \frac{b(6c^2d-e)e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + dex^2 (a+b\text{csch}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+$$

$$= \frac{b(6c^2d-e)e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 + dex^2 (a+b\text{csch}^{-1}$$

$$= \frac{b(6c^2d-e)e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 + dex^2 (a+b\text{csch}^{-1}$$

$$= \frac{b(6c^2d-e)e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 + dex^2 (a+b\text{csch}^{-1}$$

$$= \frac{b(6c^2d-e)e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 + dex^2 (a+b\text{csch}^{-1}$$

Mathematica [A] time = 0.43977, size = 148, normalized size = 0.83

$$\frac{1}{2}bd^2 \left(\text{PolyLog}\left(2, e^{-2\text{csch}^{-1}(cx)}\right) - \text{csch}^{-1}(cx)\left(\text{csch}^{-1}(cx) + 2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right)\right) + ad^2\log(x) + adex^2 + \frac{1}{4}ae^2x^4 + $$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c
*x]))/c + (b*e^2*x*(Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2) + 3*c^3*x^3*ArcCsc
h[c*x]))/(12*c^3) + a*d^2*Log[x] + (b*d^2*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2
*Log[1 - E^(-2*ArcCsch[c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]))/2
```

Maple [F] time = 0.383, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2 \left(a+b\,\mathrm{arccsch}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,ae^2x^4+4\,bc^2d^2\int \frac{x\log\left(x\right)}{4\left(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1\right)}\,dx+adex^2-bd^2\log\left(c\right)\log\left(x\right)-\frac{1}{4}\left(2\,\log\left(c^2x^2+1\right)\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + 4*b*c^2*d^2*integrate(1/4*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + a*d*e*x^2 - b*d^2*log(c)*log(x)
 - 1/4*(2*log(c^2*x^2 + 1)*log(x) + dilog(-c^2*x^2))*b*d^2 + a*d^2*log(x) +
 1/2*b*d*e*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2 - 1/24*(3*c^2*x^2 -
 2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*b*e^
2/c^4 - 1/8*(2*b*c^2*e^2*x^4*log(c) + 4*b*c^2*d^2*log(x)^2 + (8*c^2*d*e*log
(c) - e^2)*b*x^2 + 2*(b*c^2*e^2*x^4 + 4*b*c^2*d*e*x^2)*log(x) - 2*(b*c^2*e^
2*x^4 + 4*b*c^2*d*e*x^2 + 4*b*c^2*d^2*log(x))*log(sqrt(c^2*x^2 + 1) + 1))/c
^2 + 1/8*(4*c^2*d*e - e^2)*b*log(c^2*x^2 + 1)/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\text{arcsch}\,(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsch(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acsch}\,(cx))\left(d + ex^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b\,\text{arcsch}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x, x)
```

3.97 $\int \frac{(d+ex^2)^2(a+b\,\mathbf{csch}^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=178

$$-bde\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd}{}$$

```
[Out] (b*c*d^2*Sqrt[1 + 1/(c^2*x^2)])/(4*x) + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*
c) - (b*c^2*d^2*ArcCsch[c*x])/4 + b*d*e*ArcCsch[c*x]^2 - (d^2*(a + b*ArcCsc
h[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsch[c*x]))/2 - 2*b*d*e*ArcCsch[c*x]*
Log[1 - E^(2*ArcCsch[c*x])] + 2*b*d*e*ArcCsch[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsch[c*x])*Log[x^(-1)] - b*d*e*PolyLog[2, E^(2*ArcCsch[c*x])]
```

Rubi [A] time = 0.424422, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6304, 266, 43, 5789, 12, 6742, 264, 321, 215, 2325, 5659, 3716, 2190, 2279, 2391}

$$-bde\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```


```
[Out] (b*c*d^2*Sqrt[1 + 1/(c^2*x^2)])/(4*x) + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*
c) - (b*c^2*d^2*ArcCsch[c*x])/4 + b*d*e*ArcCsch[c*x]^2 - (d^2*(a + b*ArcCsc
h[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsch[c*x]))/2 - 2*b*d*e*ArcCsch[c*x]*
Log[1 - E^(2*ArcCsch[c*x])] + 2*b*d*e*ArcCsch[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsch[c*x])*Log[x^(-1)] - b*d*e*PolyLog[2, E^(2*ArcCsch[c*x])]
```


Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5789

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2 \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right.$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\,\text{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\right.}{}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\,\text{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\right.}{}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\,\text{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\right.}{}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\,\text{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\right.}{}$$

Mathematica [A] time = 0.902077, size = 187, normalized size = 1.05

$$\frac{1}{4}\left(4bde\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)-\frac{2ad^2}{x^2}+8ade\log(x)+2ae^2x^2-\frac{bd^2\left(-c^2x^2+c^2x^2\sqrt{c^2x^2+1}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)-1\right.}{cx^3\sqrt{\frac{1}{c^2x^2}+1}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsch[c*x])/x^2 + (2*b*e^2*x*(Sq
rt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c*x]))/c - (b*d^2*(-1 - c^2*x^2 + c^2*x^2
*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]]))/(c*Sqrt[1 + 1/(c^2*x^2)]*x^
3) - 4*b*d*e*ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])]) +
 8*a*d*e*Log[x] + 4*b*d*e*PolyLog[2, E^(-2*ArcCsch[c*x])])/4
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2\left(a+b\text{arccsch}\left(cx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,bc^2de\int \frac{x\log\left(x\right)}{2\left(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1\right)}\,dx-\frac{1}{2}\,be^2x^2\log\left(c\right)-\frac{1}{2}\,be^2x^2\log\left(x\right)+\frac{1}{2}\,ae^2x^2-2\,bde\log\left(c\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 4*b*c^2*d*e*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + s
qrt(c^2*x^2 + 1) + 1), x) - 1/2*b*e^2*x^2*log(c) - 1/2*b*e^2*x^2*log(x) + 1
/2*a*e^2*x^2 - 2*b*d*e*log(c)*log(x) - b*d*e*log(x)^2 - 1/2*(2*log(c^2*x^2
+ 1)*log(x) + dilog(-c^2*x^2))*b*d*e + 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2)
 + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) +
 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2) + 2*a
*d*e*log(x) + 1/4*b*e^2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2 + 1/4*
b*e^2*log(c^2*x^2 + 1)/c^2 + 1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*log(sqrt(c^2*
x^2 + 1) + 1) - 1/2*a*d^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\text{arcsch}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsch(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acsch}\,(cx))\left(d + ex^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2 (b\,\text{arcsch}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^3, x)
```

3.98 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=512

$$-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\mathrm{Po}}{}$$

```
[Out] (x*(a + b*ArcCsch[c*x]))/e + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e) + (Sq
rt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] -
Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 +
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) +
(Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^(3/2))
 - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c
*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^(3/2))
 + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e])])/(2*e^(3/2))
```

Rubi [A] time = 1.19779, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6304, 5791, 5661, 266, 63, 208, 5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\mathrm{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/e + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e) + (Sq
rt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] -
Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 +
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) +
(Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^(3/2))
```

```
 - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c
*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^(3/2))
 + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e])])/(2*e^(3/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2} \, dx = -\text{Subst}\left(\int \frac{a + b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2 \left(e + dx^2\right)} \, dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a + b\sinh^{-1}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a + b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e + dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2} \, dx, x, \frac{1}{x}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2} \, dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}} \, dx, x, \frac{1}{x}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\sinh}{2\sqrt{e}\left(\sqrt{e}+\right.}\right.\right.}{e}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x} \, dx, x, \frac{1}{x}\right)}{2e^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x} \, dx, x, \right.}{2e^{3/2}}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)} \, dx, x, \text{csch}^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{(a+bx)\text{c}}{\frac{\sqrt{e}}{c}+\sqrt{-d}\,\text{s}}\right.}{}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1 + \frac{1}{c^2x^2}}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x} \, dx, x, \text{csch}^{-1}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1 + \frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}\left(a + b\text{csch}^{-1}(cx)\right)\log\left(1 - \frac{c\sqrt{-d}e^{\text{csch}^{-}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1 + \frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}\left(a + b\text{csch}^{-1}(cx)\right)\log\left(1 - \frac{c\sqrt{-d}e^{\text{csch}^{-}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1 + \frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}\left(a + b\text{csch}^{-1}(cx)\right)\log\left(1 - \frac{c\sqrt{-d}e^{\text{csch}^{-}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2e^{3/2}}$$

Mathematica [C] time = 1.61707, size = 1221, normalized size = 2.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x + 4*b*c*Sqrt[e]*x*ArcCsch[c*x] - 4*a*c*Sqrt[d]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] + b*c*Sqrt[d]*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*
(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d
]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - b*c*Sqrt[d]*Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqrt
[d]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
b*c*Sqrt[d]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqr
t[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*c*Sq
rt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[Sqrt[e
] - (I*Sqrt[d])/x] - b*c*Sqrt[d]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - 4*b*Sqrt
[e]*Log[Tanh[ArcCsch[c*x]/2]] + (2*I)*b*c*Sqrt[d]*PolyLog[2, ((-I)*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*Po
lyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
(2*I)*b*c*Sqrt[d]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(4*c*e^(3/2))
```

Maple [F] time = 1.688, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b\operatorname{arccsch}(cx))}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\,\text{arcsch}\,(cx)+ax^2}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(a+b\,\text{acsch}\,(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d), x)
```

3.99 $\int \frac{x\left(a+b\,\textbf{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=467

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}}}{\sqrt{e-c^2d}+}\right.}{2e}$$

```
[Out] -((a + b*ArcCsch[c*x])^2/(b*e)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCs
ch[c*x])])/e + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(S
qrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcC
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] + Sqrt[-(c^2*d) + e])])/(2*e) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*
e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) +
 e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x]
)/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e)
```

Rubi [A] time = 1.10582, antiderivative size = 449, normalized size of antiderivative = 0.96, number of steps used = 26, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6304, 5791, 5659, 3716, 2190, 2279, 2391, 5799, 5561}

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}}}{\sqrt{e-c^2d}+}\right.}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1
 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e) + ((a
 + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^
2*d) + e])])/(2*e) - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e +
 (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])
)])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2
```

```
*d) + e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(S
qrt[e] + Sqrt[-(c^2*d) + e])])/(2*e) - (b*PolyLog[2, E^(2*ArcCsch[c*x])])/(
2*e)
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = -\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int\left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\operatorname{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{\operatorname{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e} + \frac{d\operatorname{Subst}\left(\int\left(-\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\sinh \ldots\right.}{2d\left(\sqrt{e}+\ldots\right.}\right.\right.}{e}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be} + \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e} - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x} \ldots\right.}{2e}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{e} + \frac{b\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, \ldots\right.}{e}$$

$$= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{e} + \frac{b\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2e} - \frac{\sqrt{-d}\operatorname{Sub}\ldots}{\ldots}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{(a \ldots}{\ldots}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{(a \ldots}{\ldots}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{(a \ldots}{\ldots}$$

Mathematica [C] time = 0.340055, size = 1103, normalized size = 2.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (b*Pi^2 - (4*I)*b*Pi*ArcCsch[c*x] - 8*b*ArcCsch[c*x]^2 + 16*b*ArcSin[Sqrt[1
 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2
*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]/(c
*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c
*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])
] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*
Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (2*I)*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqrt
[e] + (I*Sqrt[d])/x] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*
x])] + 4*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(8*e)
```

Maple [F] time = 0.552, size = 0, normalized size = 0.

$$\int \frac{x\left(a + b\operatorname{arccsch}(cx)\right)}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{x\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{ex^2+d}\,dx+\frac{a\log\left(ex^2+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d), x) + 1/2*a*
log(e*x^2 + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx\,\text{arcsch}\,(cx)+ax}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a+b\,\text{acsch}\,(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d), x)
```

3.100 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=477

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}}}{\sqrt{e-c^2d}+}\right.}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 0.849101, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6294, 5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}}}{\sqrt{e-c^2d}+}\right.}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(
```

```
Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 6294

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} -$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+bc}{}$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+bc}{}$$

$$= \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+bc}{}$$

Mathematica [C] time = 0.492968, size = 1055, normalized size = 2.21

$$4a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+8ib\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{e}}{c\sqrt{d}}+1}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(2i\text{csch}^{-1}(cx)+\pi\right)\right)}{\sqrt{e-c^2d}}\right)+8ib\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{\left(\sqrt{d}c+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(2i\text{cs}\right.\right.}{\sqrt{e-c^2d}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

```
[Out] (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4
])/Sqrt[-(c^2*d) + e]] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x]
)/(c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] - b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
 b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - (2
*I)*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + (2*I)*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + (2*I)*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(4*Sqrt[d]*Sqrt[e])
```

Maple [F] time = 1.143, size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{arccsch}\,(cx)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsch}(cx)}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

3.101 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=425

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\ldots}\right)}{2d}$$

```
[Out] (a + b*ArcCsch[c*x])^2/(2*b*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -((c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d) - (b*PolyLog
[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])
/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
 + e])])/(2*d)
```

Rubi [A] time = 0.862468, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6304, 5791, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\ldots}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a + b*ArcCsch[c*x])^2/(2*b*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -((c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d) - (b*PolyLog
[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])
```

```
/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
 + e])])/(2*d)
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-de}\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{}}{\sqrt{}}\right.}{2d}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{}}{\sqrt{}}\right.}{2d}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{}}{\sqrt{}}\right.}{2d}$$

Mathematica [C] time = 0.957291, size = 387, normalized size = 0.91

$$b\left(\text{PolyLog}\left(2, \frac{\left(2\sqrt{e\left(e-c^2d\right)}+c^2d-2e\right)e^{-2\text{csch}^{-1}(cx)}}{c^2d}\right)+\text{PolyLog}\left(2, \frac{\left(c^2d-2\left(\sqrt{e\left(e-c^2d\right)}+e\right)\right)e^{-2\text{csch}^{-1}(cx)}}{c^2d}\right)-2\left(i\sin^{-1}\left(\sqrt{\frac{e}{c^2d}}\right)\left(2\tanh^{-1}\left(\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*(-2*(ArcCsch[c*x]^2 + I*ArcSin[Sqrt[e/
(c^2*d)]]*(2*ArcTanh[Sqrt[e*(-(c^2*d) + e)]/(c*e*Sqrt[1 + 1/(c^2*x^2)]*x)]
- Log[(2*e - 2*Sqrt[e*(-(c^2*d) + e)] + c^2*d*(-1 + E^(2*ArcCsch[c*x])))/(c
^2*d*E^(2*ArcCsch[c*x]))] + Log[(2*(e + Sqrt[e*(-(c^2*d) + e)]) + c^2*d*(-1
 + E^(2*ArcCsch[c*x])))/(c^2*d*E^(2*ArcCsch[c*x]))]) + ArcCsch[c*x]*(Log[(2
*e - 2*Sqrt[e*(-(c^2*d) + e)] + c^2*d*(-1 + E^(2*ArcCsch[c*x])))/(c^2*d*E^(
2*ArcCsch[c*x]))] + Log[(2*(e + Sqrt[e*(-(c^2*d) + e)]) + c^2*d*(-1 + E^(2*
ArcCsch[c*x])))/(c^2*d*E^(2*ArcCsch[c*x]))])) + PolyLog[2, (c^2*d - 2*e + 2
*Sqrt[e*(-(c^2*d) + e)])/(c^2*d*E^(2*ArcCsch[c*x]))] + PolyLog[2, (c^2*d -
2*(e + Sqrt[e*(-(c^2*d) + e)]))/(c^2*d*E^(2*ArcCsch[c*x]))]))/(4*d)
```

Maple [F] time = 0.48, size = 0, normalized size = 0.

$$\int \frac{a + b\mathrm{arccsch}\,(cx)}{x\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{\log\left(ex^2 + d\right)}{d} - \frac{2\,\log\left(x\right)}{d}\right) + b\int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{ex^3 + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c^2*x^2) +
 1) + 1/(c*x))/(e*x^3 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcsch}\,(cx)+a}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{acsch}\,(cx)}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arcsch}\,(cx)+a}{\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x), x)
```

3.102 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=518

$$-\frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\mathrm{PolyLog}}{2}$$

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcCsch[c*x])/(d*x) + (Sqrt[e]
*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) + (
Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/
2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyL
og[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*(-
d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sq
rt[-(c^2*d) + e])])/(2*(-d)^(3/2))
```

Rubi [A] time = 1.05984, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6304, 5791, 5653, 261, 5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\mathrm{PolyLog}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```



```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcCsch[c*x])/(d*x) + (Sqrt[e]
*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) + (
Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
```

```
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/
2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyL
og[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*(-
d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sq
rt[-(c^2*d) + e])])/(2*(-d)^(3/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right) dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\,\text{Subst}\left(\int \sinh^{-1}\left(\frac{x}{c}\right) dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d} + \frac{\sqrt{e}\,\text{Su}}{}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\,\text{Subst}\,(}{}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\,\text{Sub}}{}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}\left(a+b\text{cs}\right.}{}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}\left(a+b\text{cs}\right.}{}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}\left(a+b\text{cs}\right.}{}$$

Mathematica [C] time = 1.60399, size = 1211, normalized size = 2.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[1
 + 1/(c^2*x^2)] - ArcCsch[c*x]/x)/d - ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcC
sch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1
 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsc
h[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e
] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/d^(3/2)
+ ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*Arc
Sin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Co
t[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1
- E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^
(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])]))/d^(3/2))
```

Maple [F] time = 1.497, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x^2\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcsch}\,(cx)+a}{ex^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{acsch}\,(cx)}{x^2\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x^2), x)
```

3.103 $\int \frac{x^5\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=591

$$-\frac{bd\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{e^3}-\frac{bd\text{PolyLog}\left(2,\frac{c\sqrt{-d}}{\sqrt{e}}\right.}{e^3}$$

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsch[c*x]))/(2*e^2*(e
+ d/x^2)) + (x^2*(a + b*ArcCsch[c*x]))/(2*e^2) + (2*d*(a + b*ArcCsch[c*x])^
2)/(b*e^3) - (b*d*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x
)])/(2*Sqrt[c^2*d - e]*e^(5/2)) + (2*d*(a + b*ArcCsch[c*x])*Log[1 - E^(-2*A
rcCsch[c*x])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x
])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, E^(-2*ArcCsch[c*x
])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(
c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3
```

Rubi [A] time = 1.28155, antiderivative size = 571, normalized size of antiderivative = 0.97, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6304, 5791, 5661, 264, 5659, 3716, 2190, 2279, 2391, 5787, 377, 205, 5799, 5561}

$$-\frac{bd\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{e^3}-\frac{bd\text{PolyLog}\left(2,\frac{c\sqrt{-d}}{\sqrt{e}}\right.}{e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsch[c*x]))/(2*e^2*(e
+ d/x^2)) + (x^2*(a + b*ArcCsch[c*x]))/(2*e^2) - (b*d*ArcTan[Sqrt[c^2*d - e
]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d - e]*e^(5/2)) - (d*(a
 + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^
2*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 -
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3 - (d*(a +
b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e])])/e^3 + (2*d*(a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e^3
- (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) +
e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(
c^2*d) + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3 + (b*d*PolyLog[2, E^(2*ArcCsch[c*x])]
)/e^3
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} + \frac{2d^2x\left(a\right.}{e^3}\right.\right.$$

$$= \frac{(2d)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} + \frac{(2d)\,\text{Subst}\left(\int(a+bx)\coth(x)\,dx, x, \text{csch}^{-1}\right.}{e^3}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^3} - \frac{(-d)}{}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^3} - \frac{bd\,\text{ta}}{}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{5/2}} + \frac{2d}{}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{5/2}} - \frac{d}{}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{5/2}} - \frac{d}{}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{5/2}} - \frac{d}{}$$

Mathematica [C] time = 6.01723, size = 1447, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(d*Pi^2 - (
2*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsch[c*x] - 2*e*x^2*ArcCsch[
c*x] + (d^(3/2)*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcCsch[c*
x])/(Sqrt[d] + I*Sqrt[e]*x) - 8*d*ArcCsch[c*x]^2 - 2*d*ArcSinh[1/(c*x)] + 1
6*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt
[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*d*ArcSin[Sq
rt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*d*ArcCsch[c*x]*Log[1 - E^
(-2*ArcCsch[c*x])] + (2*I)*d*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[1 + Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[1 -
Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sqrt[1
- Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e]
 + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sqrt[1
 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*d*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2
*I)*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(
I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/
(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e] + (d*Sqrt
[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*
Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))])/S
qrt[-(c^2*d) + e] + 4*d*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*d*PolyLog[2, ((
-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*Poly
Log[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*
d*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] + 4*d*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])]))/(4*e^3)
```

Maple [F] time = 0.585, size = 0, normalized size = 0.

$$\int \frac{x^5 (a + b\,\mathrm{arccsch}\,(cx))}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{d^2}{e^4x^2 + de^3} - \frac{x^2}{e^2} + \frac{2\,d\log\left(ex^2 + d\right)}{e^3}\right) + b\int \frac{x^5\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^4 + 2\,dex^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^5\,\mathrm{arcsch}\,(cx) + ax^5}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsch(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=553

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}}}{\sqrt{e-c^2d}+\sqrt{\ }}\right.}{2e^2}$$

[Out] -(a + b*ArcCsch[c*x])/(2*e*(e + d/x^2)) - (a + b*ArcCsch[c*x])^2/(b*e^2) + (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d - e]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[c*x])])/e^2 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2)

Rubi [A] time = 1.24854, antiderivative size = 535, normalized size of antiderivative = 0.97, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6304, 5791, 5659, 3716, 2190, 2279, 2391, 5787, 377, 205, 5799, 5561}

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}}}{\sqrt{e-c^2d}+\sqrt{\ }}\right.}{2e^2}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]

[Out] -(a + b*ArcCsch[c*x])/(2*e*(e + d/x^2)) + (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d - e]*e^(3/2)) + ((a + b*ArcCsc

```
h[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]
)/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt
[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*Arc
Csch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])])/(2*e^2) - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e^2 + (b*
PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/
(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d
) + e])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/
(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2) - (b*PolyLog[2, E^(2*ArcCsch[c*x])
])/(2*e^2)
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^2x} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{x(a+b\cdots}{(e\cdots}\right.}{\cdots}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\operatorname{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int\left(-\frac{\sqrt{-d}(a+b\cdots}{2d(\sqrt{e}\cdots}\right.\right.}{\cdots}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be^2} + \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^2} - \frac{\sqrt{-d}\,\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be^2} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\cdots}{e^2}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{e^2} + \frac{b\,\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \cdots$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \cdots$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \cdots$$

Mathematica [C] time = 2.32482, size = 1410, normalized size = 2.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsch[c*x] + (2*b*Sqrt[d]*ArcC
sch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] + I
*Sqrt[e]*x) - 8*b*ArcCsch[c*x]^2 - 4*b*ArcSinh[1/(c*x)] + 16*b*ArcSin[Sqrt[
1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (
2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[
c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x]
)] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]
/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] - (2*I)*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqr
t[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*
(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e] + (2*b*Sqrt[e]*Log[(-2*
Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(
c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
 + e] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*b*Poly
Log[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])])/(8*e^2)
```

Maple [F] time = 0.476, size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b\operatorname{arccsch}(cx))}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{d}{e^3x^2 + de^2} + \frac{\log\left(ex^2 + d\right)}{e^2}\right) + b\int \frac{x^3 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^4 + 2\,dex^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqrt
(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^3 \operatorname{arcsch}(cx) + ax^3}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

3.105 $\int \frac{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bcx\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{2de\sqrt{-c^2x^2}}+\frac{bcx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{e}\sqrt{-c^2x^2}\sqrt{c^2d-e}}$$

```
[Out] -(a + b*ArcCsch[c*x])/(2*e*(d + e*x^2)) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2]]
)/(2*d*e*Sqrt[-(c^2*x^2)]) + (b*c*x*ArcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sq
rt[c^2*d - e]])/(2*d*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.150264, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6300, 446, 86, 63, 205, 208}

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bcx\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{2de\sqrt{-c^2x^2}}+\frac{bcx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{e}\sqrt{-c^2x^2}\sqrt{c^2d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCsch[c*x])/(2*e*(d + e*x^2)) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2]]
)/(2*d*e*Sqrt[-(c^2*x^2)]) + (b*c*x*ArcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sq
rt[c^2*d - e]])/(2*d*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[-(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)}\,dx, x, x^2\right)}{4e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}(d+ex)}\,dx, x, x^2\right)}{4d\sqrt{-c^2x^2}} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}}\,dx, x,\right.}{4de\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{2cd\sqrt{-c^2x^2}} - \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x\right.}{2cde\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{bcx\tan^{-1}\left(\sqrt{-1-c^2x^2}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\sqrt{e}\sqrt{-c^2x^2}}$$

Mathematica [C] time = 0.719456, size = 271, normalized size = 1.95

$$-\frac{\frac{2a}{d+ex^2} + \frac{b\sqrt{e}\log\left(-\frac{4\left(cd\sqrt{e}x\left(c\sqrt{d}+i\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}\right)+ide\right)}{b\sqrt{e-c^2d}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{e-c^2d}} + \frac{b\sqrt{e}\log\left(\frac{4i\left(de+cd\sqrt{e}x\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}+ic\sqrt{d}\right)\right)}{b\sqrt{e-c^2d}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{e-c^2d}} + \frac{2b\operatorname{csch}^{-1}(cx)}{d+ex^2} - \frac{2b\sinh^{-1}\left(\frac{1}{cx}\right)}{d}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -((2*a)/(d + e*x^2) + (2*b*ArcCsch[c*x])/(d + e*x^2) - (2*b*ArcSinh[1/(c*x)
])/d + (b*Sqrt[e]*Log[(-4*(I*d*e + c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^2*d)
 + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^2*d) + e]*(Sqrt[d] - I*Sqrt[e]
*x))])/(d*Sqrt[-(c^2*d) + e]) + (b*Sqrt[e]*Log[((4*I)*(d*e + c*d*Sqrt[e]*(I
*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^2*d)
 + e]*(Sqrt[d] + I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) + e]))/(4*e)
```

Maple [B] time = 0.273, size = 358, normalized size = 2.6

$$-\frac{ac^{2}}{2e\left(c^{2}x^{2}e+c^{2}d\right)}-\frac{bc^{2}\operatorname{arccsch}(cx)}{2e\left(c^{2}x^{2}e+c^{2}d\right)}+\frac{b}{2cxed}\sqrt{c^{2}x^{2}+1}\operatorname{Artanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)\frac{1}{\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}-\frac{b}{4cxed}\sqrt{c^{2}x^{2}+1}\ln\left(2\frac{}{cxe+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccsch(c*x)+1
/2/c*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d*arctanh(1/(c^2*x
^2+1)^(1/2))-1/4/c*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-
(c^2*d-e)/e)^(1/2)*ln(2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e-(-c^2*d*e
)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))-1/4/c*b/e*(c^2*x^2+1)^(1/2)/((c^2*
x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)*ln(-2*((-(c^2*d-e)/e)^(1/2)*
(c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\left(4c^{2}\int\frac{x}{2\left(c^{2}e^{2}x^{4}+\left(c^{2}de+e^{2}\right)x^{2}+de+\left(c^{2}e^{2}x^{4}+\left(c^{2}de+e^{2}\right)x^{2}+de\right)\sqrt{c^{2}x^{2}+1}\right)}\,dx-\frac{2c^{2}d^{2}\log(c)-2\left(c^{2}de-e^{2}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^2*integrate(1/2*x/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e + (c^2
*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(c^2*x^2 + 1)), x) - (2*c^2*d^2*l
og(c) - 2*(c^2*d*e - e^2)*x^2*log(x) - 2*d*e*log(c) + (c^2*d*e*x^2 + c^2*d^
2)*log(c^2*x^2 + 1) - 2*(c^2*d^2 - d*e)*log(sqrt(c^2*x^2 + 1) + 1))/(c^2*d^
3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2) + log(e*x^2 + d)/(c^2*d^2 - d*e)
)*b - 1/2*a/(e^2*x^2 + d*e)
```

Fricas [B] time = 2.73245, size = 1288, normalized size = 9.27

$$\left[-\frac{2\,ac^2d^2-2\,ade+\sqrt{-c^2de+e^2}\left(bex^2+bd\right)\log\left(\frac{c^2ex^2-c^2d-2\,\sqrt{-c^2de+e^2}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,e}{ex^2+d}\right)-2\left(bc^2d^2-bde+\left(bc^2de-be^2\right)x^2\right)\log}{4\left(c^2d^3e-d^2e^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*a*d*e + sqrt(-c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^
2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 2*e)/(e*x^2 + d)) - 2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^2*d^2 - b*d*e + (b*c^2
*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*
c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d
^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*e + sqrt
(c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x
^2 + 1)/(c^2*x^2))/(c^2*d - e)) - (b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*
x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^2 - b*d*e
+ (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1)
 + (b*c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/
(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^2, x)
```

3.106 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal. Leaf size=515

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\dots}\right)}{2d^2}$$

```
[Out] -(e*(a + b*ArcCsch[c*x]))/(2*d^2*(e + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2*b
*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*
x)])/(2*d^2*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^
ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d^2) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
+ Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]
*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^2) - (b*PolyLog[2, -
((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^2) - (b
*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + S
qrt[-(c^2*d) + e])])/(2*d^2)
```

Rubi [A] time = 1.15307, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6304, 5791, 5787, 377, 205, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\dots}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -(e*(a + b*ArcCsch[c*x]))/(2*d^2*(e + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2*b
*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*
```

```
x)])/(2*d^2*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^
ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d^2) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
+ Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]
*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^2) - (b*PolyLog[2, -
((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^2) - (b
*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + S
qrt[-(c^2*d) + e])])/(2*d^2)
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\operatorname{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int \left(-\frac{ex\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\operatorname{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}-\frac{\operatorname{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d}+\frac{(be)\operatorname{Su}\cdots}{\cdots}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}}-\frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}}+\cdots$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}+\frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}\cdots$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}+\frac{\operatorname{Subst}\left(\int \frac{e^x(a+\cdots}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+\cdots}}{c}\cdots}\cdots\right)}{2\cdots}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{l}\cdots}{2\cdots}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{l}\cdots}{2\cdots}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{l}\cdots}{2\cdots}$$

Mathematica [F] time = 43.2071, size = 0, normalized size = 0.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2), x]
```

Maple [F] time = 0.586, size = 0, normalized size = 0.

$$\int \frac{a + b\,\text{arccsch}\,(cx)}{x\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{1}{dex^2 + d^2} - \frac{\log\left(ex^2 + d\right)}{d^2} + \frac{2\,\log\left(x\right)}{d^2}\right) + b\int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^5 + 2\,dex^3 + d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{e^2x^5 + 2\,dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x), x)
```

3.107 $\int \frac{x^4 \left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=756

$$-\frac{3b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}-\frac{3b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-}}{}$$

```
[Out] -(d*(a + b*ArcCsch[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcC
sch[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsch[c*x]))/e^2
+ (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d -
(Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(
4*Sqrt[c^2*d - e]*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/
(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[c^2*d - e]*e^2)
 + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sq
rt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsch[c*x]
)*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e
^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCs
ch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])
])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sq
rt[e] - Sqrt[-(c^2*d) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*b*
Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2))
```

Rubi [A] time = 2.46837, antiderivative size = 756, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6304, 5791, 5661, 266, 63, 208, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$-\frac{3b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}-\frac{3b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(d*(a + b*ArcCsch[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcC
sch[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsch[c*x]))/e^2
+ (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d -
(Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(
4*Sqrt[c^2*d - e]*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/
(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[c^2*d - e]*e^2)
 + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sq
rt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsch[c*x]
)*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e
^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCs
ch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])
])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sq
rt[e] - Sqrt[-(c^2*d) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*b*
Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int\left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}}{(e+dx^2}\right.}{e}$$

$$= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\operatorname{Subst}\left(\int\left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\sinh^{-1}}{2\sqrt{e}\left(\sqrt{e}+\sqrt{}\right.}\right.\right.}{e^2}$$

$$= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{(a+bx)\operatorname{co}}{\frac{\sqrt{e}}{c}-\sqrt{-d}\operatorname{s}}\right.}{2}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right.}{ce^2}$$

Mathematica [C] time = 6.23543, size = 1583, normalized size = 2.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*e^(5/2)) + b*(-(d*(-(ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) -
(I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqr
t[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*
(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(4*e^2) - (d*(-(Ar
cCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - L
og[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[
1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(
c^2*d) + e]))/Sqrt[d]))/(4*e^2) - (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I)*Pi*ArcC
sch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1
 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsc
h[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e
] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/e^(5/2)
+ (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Lo
g[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - S
qrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2
, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])
```

```
*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/e^(5/2) + ((ArcCsch[c*x]*Coth[ArcCsch[c*
x]/2])/2 - Log[Tanh[ArcCsch[c*x]/2]] - (ArcCsch[c*x]*Tanh[ArcCsch[c*x]/2])/
2)/(c*e^2))
```

Maple [F] time = 13.253, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b\operatorname{arccsch}(cx))}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arcsch}(cx) + ax^4}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcsch}(cx) + a)x^4}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

3.108 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=719

$$-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\cdots}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[Out] (a + b*ArcCsch[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsch[c*x])/
(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/
(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d -
 e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d - e]*e) + ((a + b*ArcCsch
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])
/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcC
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*Sqr
t[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLo
g[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-
d]*e^(3/2))
```

Rubi [A] time = 1.24176, antiderivative size = 719, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6304, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\cdots}\right)}{4\sqrt{-d}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCsch[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsch[c*x])/
(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/
(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d -
 e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d - e]*e) + ((a + b*ArcCsch
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])
/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcC
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*Sqr
t[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLo
g[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-
d]*e^(3/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x,\right.$$

$$= \frac{d\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin}{-de-}\right.}{2e}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce} + \frac{b\,\text{Subst}\left(\right.}{}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\sin}{\sqrt{e}+}\right.}{4e^{3}}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{b\tanh^{-1}\left(\frac{c^2d+}{c\sqrt{d}\sqrt{c^2d-}}\right)}{4\sqrt{d}\sqrt{c^2d-}}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{b\tanh^{-1}\left(\frac{c^2d+}{c\sqrt{d}\sqrt{c^2d-}}\right)}{4\sqrt{d}\sqrt{c^2d-}}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{b\tanh^{-1}\left(\frac{c^2d+}{c\sqrt{d}\sqrt{c^2d-}}\right)}{4\sqrt{d}\sqrt{c^2d-}}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{b\tanh^{-1}\left(\frac{c^2d+}{c\sqrt{d}\sqrt{c^2d-}}\right)}{4\sqrt{d}\sqrt{c^2d-}}$$

$$= \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{b\tanh^{-1}\left(\frac{c^2d+}{c\sqrt{d}\sqrt{c^2d-}}\right)}{4\sqrt{d}\sqrt{c^2d-}}$$

<u>Mathematica</u> [C] time = 2.57977, size = 1442, normalized size = 2.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((2*ArcCsch[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsch[c*x])/(I*Sqrt[d]
+ Sqrt[e]*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[
((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]
])/Sqrt[d] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]])
/Sqrt[d] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])]])/Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - (4*ArcSin[Sqrt[1 + Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - ((2*I)*ArcCsch[c*x]*Log[1 + (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (4
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (Pi*Log[1 - (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - ((2*I)*Ar
cCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])]])/Sqrt[d] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 -
(I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - (
Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/
Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])]])/Sqrt[d] - (Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]])/Sqrt[d] + (Pi*Log[Sqrt[
e] + (I*Sqrt[d])/x]])/Sqrt[d] + ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqr
t[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt
[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]])/(Sqrt[d]*Sqrt[-(c^2*d) + e]) - ((
2*I)*Sqrt[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e
]*x))]])/(Sqrt[d]*Sqrt[-(c^2*d) + e]) - ((2*I)*PolyLog[2, ((-I)*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + ((2*I)*PolyLog[
2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d]
 + ((2*I)*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])]])/Sqrt[d] - ((2*I)*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d]))/(8*e^(3/2))
```

Maple [F] time = 1.7, size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(a +b \,\mathrm{arccsch}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b x^{2} \operatorname{arcsch}\left(c x\right) + a x^{2}}{e^{2} x^{4} + 2 \, d e x^{2} + d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^2}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

3.109 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=713

$$\frac{b\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}}}{\sqrt{e-c^2d}+}\right.}{4(-d)^{3/2}\sqrt{e}}$$

```
[Out] -(a + b*ArcCsch[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsch[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])]])/(4*d^(3/2)*Sqrt[c^2*d
- e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e
]*Sqrt[1 + 1/(c^2*x^2)])]])/(4*d^(3/2)*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*
x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]])/(4
*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*Arc
Csch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])]])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])]])/(4*(-d)^(3/2)*Sqrt[e]) + (b*P
olyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))]])/(
4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e])]])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d
]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))]])/(4*(-d)^(3/2)*Sqrt[e])
- (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])]
)/(4*(-d)^(3/2)*Sqrt[e])
```

Rubi [A] time = 2.18736, antiderivative size = 713, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6294, 5791, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{csch}}}{\sqrt{e-c^2d}+}\right.}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCsch[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsch[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d
- e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e
]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*
x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4
*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*Arc
Csch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*P
olyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(
4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d
]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e])
- (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])]
)/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 6294

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
```

```
0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = -\mathrm{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\mathrm{Subst}\left(\int\left(-\frac{e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\mathrm{Subst}\left(\int\left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\mathrm{Subst}\left(\int\left(-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\sinh\right.}{4e\left(\sqrt{-d}\sqrt{e}\right.}\right.}{d}$$

$$= -\left(\frac{1}{4}\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right)-\frac{1}{4}\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)-\frac{1}{2}\mathrm{Subs}$$

$$= -\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{1}{2}\mathrm{Subst}\left(\int\left(-\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\sinh^{-1}}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{\right.}}\right.\right.$$

$$= -\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cd}+\frac{b\,\mathrm{Subst}\left(\int \frac{}{d^2}\right.}{}$$

$$= -\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-}}{}}{c\sqrt{d}\sqrt{c^2d-e}}\right.}{4d^{3/2}\sqrt{c^2d-e}}$$

$$= -\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-}}{}}{c\sqrt{d}\sqrt{c^2d-e}}\right.}{4d^{3/2}\sqrt{c^2d-e}}$$

$$= -\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\mathrm{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-}}{}}{c\sqrt{d}\sqrt{c^2d-e}}\right.}{4d^{3/2}\sqrt{c^2d-e}}$$

$$b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{}}}\right)\quad b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-}}{}}{c\sqrt{d}\sqrt{c^2d-e}}\right.$$

Mathematica [C] time = 3.48739, size = 1437, normalized size = 2.02

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((2*Sqrt[d]*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*Ar
cCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*S
qrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x
])/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])
/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + ((2*I)*ArcCsch[c*x]*Log[1 -
 (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] -
 (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + (Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] - ((2*I
)*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])]])/Sqrt[e] + (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[
e] + (Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])]])/Sqrt[e] - ((2*I)*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])]])/Sqrt[e] - (Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + ((2*I)*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + (4*ArcSin[Sqr
t[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] - (Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]
)/Sqrt[e] + (Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/Sqrt[e] - ((2*I)*Log[(2*Sqrt[
d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2
*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]])/Sqrt[-(c^2*d) +
e] + ((2*I)*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]
*x))]])/Sqrt[-(c^2*d) + e] - ((2*I)*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + ((2*I)*PolyLog[2, (I*(-Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + ((2*I)*P
olyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
)/Sqrt[e] - ((2*I)*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])]])/Sqrt[e]))/(4*d^(3/2)))/2
```

Maple [F] time = 1.997, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\left(cx\right)}{\left(e x^{2}+d\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcsch}\left(cx\right)+a}{e^{2}x^{4}+2\,dex^{2}+d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^2, x)
```

3.110 $\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=758

$$\frac{3b\sqrt{e}\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\text{Poly}\ldots}{}$$

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcCsch[c*x])/(d^2*x) + (e
*(a + b*ArcCsch[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsch
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*
Sqrt[c^2*d - e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d - e]) - (3*Sq
rt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/
2)) - (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/
(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqr
t[e] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (
3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*
d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2))
```

Rubi [A] time = 2.24642, antiderivative size = 758, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {6304, 5791, 5653, 261, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{3b\sqrt{e}\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\text{Poly}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcCsch[c*x])/(d^2*x) + (e
*(a + b*ArcCsch[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsch
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*
Sqrt[c^2*d - e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d - e]) - (3*Sq
rt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/
2)) - (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/
(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqr
t[e] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (
3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*
d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
```

```
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d^2} + \frac{e^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} - \frac{2e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{a}{d^2x} - \frac{b\,\text{Subst}\left(\int \sinh^{-1}\left(\frac{x}{c}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \ldots\right.}{d^2}$$

$$= -\frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{\sqrt{e}\ldots}{\ldots}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\sqrt{e}\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh^{-1}\left(\ldots\right.}{4d^5\ldots}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh^{-1}\left(\ldots\right.}{4d^5\ldots}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh^{-1}\left(\ldots\right.}{4d^5\ldots}$$

$$\ldots\ be\tanh^{-1}\ldots$$

Mathematica [C] time = 2.87637, size = 1487, normalized size = 1.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(8*c*Sqrt[d]*Sqrt[1 + 1/(c^2*x^2)] - (8*Sqrt[d]*ArcCs
ch[c*x])/x - (2*Sqrt[d]*e*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2*S
qrt[d]*e*ArcCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (24*I)*Sqrt[e]*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - (24*I)*Sqrt[e]*ArcSin[Sqrt[
1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (
2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] + 3*Sqrt[e]*Pi*Log[1 - (I*(-Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 3*Sqrt[e]*P
i*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] - 3*Sqrt[e]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + 3*Sqrt[e]*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcS
in[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 3*Sqrt[e]*Pi*Log[Sqrt[e] - (I*Sqrt[
d])/x] - 3*Sqrt[e]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + ((2*I)*e*Log[(2*Sqrt[d
]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*
x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e
] - ((2*I)*e*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e
]*x))])/Sqrt[-(c^2*d) + e] + (6*I)*Sqrt[e]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*PolyLog[2, (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[
e]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + (6*I)*Sqrt[e]*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])]))/(8*d^(5/2))
```

Maple [F] time = 19.474, size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x^{2}\left(e\,x^{2}+d\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcsch}\left(cx\right)+a}{e^{2}x^{6}+2\,dex^{4}+d^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^2 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

3.111 $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=694

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}}}{\sqrt{e-c^2d}+}\right.}{2e^3}$$

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(8*(c^2*d - e)*e^2*(e + d/x^2)*x) - (a + b*Ar
cCsch[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsch[c*x])/(2*e^2*(e + d/x^2))
- (a + b*ArcCsch[c*x])^2/(b*e^3) + (b*(c^2*d - 2*e)*ArcTan[Sqrt[c^2*d - e]/
(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*(c^2*d - e)^(3/2)*e^(5/2)) + (b*Ar
cTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d -
e]*e^(5/2)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[c*x])])/e^3 + ((a
 + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^
2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[
1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) +
((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-
(c^2*d) + e])])/(2*e^3) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*e^3) + (b*
PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/
(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d
) + e])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/
(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3)
```

Rubi [A] time = 1.40827, antiderivative size = 676, normalized size of antiderivative = 0.97, number of steps used = 33, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {6304, 5791, 5659, 3716, 2190, 2279, 2391, 5787, 382, 377, 205, 5799, 5561}

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}}}{\sqrt{e-c^2d}+}\right.}{2e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(8*(c^2*d - e)*e^2*(e + d/x^2)*x) - (a + b*Ar
cCsch[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsch[c*x])/(2*e^2*(e + d/x^2))
+ (b*(c^2*d - 2*e)*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*
x)])/(8*(c^2*d - e)^(3/2)*e^(5/2)) + (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*S
qrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d - e]*e^(5/2)) + ((a + b*ArcCsch[c*x
])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*
e^3) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] -
 Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/
(2*e^3) - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e^3 + (b*PolyL
og[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*e^
3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e
])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[
-(c^2*d) + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) - (b*PolyLog[2, E^(2*ArcCsch[c*x])])/(2
*e^3)
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=-\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx,x,\frac{1}{x}\right)$$

$$=-\text{Subst}\left(\int\left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^3x}-\frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3}-\frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2}-\frac{dx\left(a+\right.}{e^3}\right.\right.$$

$$=-\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{e^3}+\frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{e^3}+\frac{d\,\text{Subst}\left(\int \frac{x\left(a+\right.}{}\right.}{}$$

$$=-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}-\frac{\text{Subst}\left(\int(a+bx)\coth(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{e^3}+\frac{d}{}$$

$$=\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be^3}+\frac{2}{}$$

$$=\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be^3}+\frac{b}{}$$

$$=\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{b\left(c^2d-2e\right)\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}\sqrt{}}\right.}{8\left(c^2d-e\right)^{3/2}e^{5/2}}$$

$$=\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{b\left(c^2d-2e\right)\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}\sqrt{}}\right.}{8\left(c^2d-e\right)^{3/2}e^{5/2}}$$

$$=\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{b\left(c^2d-2e\right)\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}\sqrt{}}\right.}{8\left(c^2d-e\right)^{3/2}e^{5/2}}$$

$$=\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{b\left(c^2d-2e\right)\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}\sqrt{}}\right.}{8\left(c^2d-e\right)^{3/2}e^{5/2}}$$

Mathematica [C] time = 7.62451, size = 2023, normalized size = 2.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(a*d^2)/(4*e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x^2
])/(2*e^3) + b*(-(d*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d
- e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sq
rt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt
[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/
(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3
/2))))/(16*e^(5/2)) - (d*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]
*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] +
Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*
d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[
1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d -
 e)^(3/2))))/(16*e^(5/2)) - (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/(I*Sqrt[d]*
Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*
Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(S
qrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/
e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x))
 + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c
*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e
]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/e^(5/2) + (Pi^2
- (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*
x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] +
 (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
(4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I
)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*P
olyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])])/(16*e^3) + (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*ArcS
in[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot
[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 -
```

```
 E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[e
]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c
^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^(
-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])])/(16*e^3))
```

Maple [F] time = 0.595, size = 0, normalized size = 0.

$$\int \frac{x^5 \left(a + b\,\mathrm{arccsch}\left(cx\right)\right)}{\left(ex^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\, a \left(\frac{4\, dex^2 + 3\, d^2}{e^5x^4 + 2\, de^4x^2 + d^2e^3} + \frac{2\, \log\left(ex^2 + d\right)}{e^3}\right) + b \int \frac{x^5 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^3x^6 + 3\, de^2x^4 + 3\, d^2ex^2 + d^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^3*x^6 +
 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^5 \operatorname{arcsch}(cx) + ax^5}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsch(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

3.112 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=167

$$\frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8de^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}}-\frac{bcx\sqrt{-c^2x^2-1}}{8e\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

```
[Out] -(b*c*x*Sqrt[-1 - c^2*x^2])/(8*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*(d + e*x^2))
+ (x^4*(a + b*ArcCsch[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d - 2*e)*x*Arc
Tanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d*(c^2*d - e)^(3/2)*
e^(3/2)*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.215472, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 6302, 12, 446, 78, 63, 208}

$$\frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8de^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}}-\frac{bcx\sqrt{-c^2x^2-1}}{8e\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```


```
[Out] -(b*c*x*Sqrt[-1 - c^2*x^2])/(8*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*(d + e*x^2))
+ (x^4*(a + b*ArcCsch[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d - 2*e)*x*Arc
Tanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d*(c^2*d - e)^(3/2)*
e^(3/2)*Sqrt[-(c^2*x^2)])
```


Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bcx)\int \frac{x^3}{4d\sqrt{-1-c^2x^2}\left(d+ex^2\right)^2}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bcx)\int \frac{x^3}{\sqrt{-1-c^2x^2}\left(d+ex^2\right)^2}\,dx}{4d\sqrt{-c^2x^2}}$$

$$= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bcx)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1-c^2x}(d+ex)^2}\,dx, x, x^2\right)}{8d\sqrt{-c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\left(c^2d-2e\right)x\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{-1\cdots}}\right.}{16d\left(c^2d-e\right)e\sqrt{-\cdots}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{\left(b\left(c^2d-2e\right)x\right)\mathrm{Subst}\left(\int \frac{1}{d-\frac{e}{c^2}\cdots}\right.}{8cd\left(c^2d-e\right)e\sqrt{\cdots}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{bc\left(c^2d-2e\right)x\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1-\cdots}}{\sqrt{c^2d-\cdots}}\right.}{8d\left(c^2d-e\right)^{3/2}e^{3/2}\sqrt{-c^2x^2}}$$

Mathematica [C] time = 1.435, size = 375, normalized size = 2.25

$$-\frac{\frac{8a}{d+ex^2} - \frac{4ad}{\left(d+ex^2\right)^2} + \frac{b\sqrt{e}\left(2e-c^2d\right)\log\left(\frac{16de^{3/2}\sqrt{e-c^2d}\left(\sqrt{e}+cx\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}-ic\sqrt{d}\right)\right)}{b\left(2e-c^2d\right)\left(\sqrt{e}x+i\sqrt{d}\right)}\right)}{d\left(e-c^2d\right)^{3/2}} + \frac{b\sqrt{e}\left(2e-c^2d\right)\log\left(-\frac{16ide^{3/2}\sqrt{e-c^2d}\left(\sqrt{e}+cx\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}+ic\sqrt{d}\right)\right)}{b\left(c^2d-2e\right)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\left(e-c^2d\right)^{3/2}} - \cdots}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 + 1/(c^2*x^2
)]*x)/((-(c^2*d) + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcCsch[c*x])/(d +
e*x^2)^2 - (4*b*ArcSinh[1/(c*x)])/d + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[(16*d
*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) +
e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(-(c^2*d) + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))]
)/(d*(-(c^2*d) + e)^(3/2)) + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[((-16*I)*d*e^(
```

```
3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt
[1 + 1/(c^2*x^2)])*x))/(b*(c^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^
2*d) + e)^(3/2)))/(16*e^2)
```

Maple [B] time = 0.281, size = 1914, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^
2*b*arccsch(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*b*arccsch(c*x)/e^2*d/(c^2*e*
x^2+c^2*d)^2-1/4*c^3*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/(c^2
*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2
+1)^(1/2))-1/4*c^3*b*(c^2*x^2+1)^(1/2)/e/((c^2*x^2+1)/c^2/x^2)^(1/2)/x*d/(c
^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x
^2+1)^(1/2))+1/16*c^3*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/(-(
c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/
2))*ln(2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/
(c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^3*b*(c^2*x^2+1)^(1/2)/e/((c^2*x^2+1)/c^2/x
^2)^(1/2)/x*d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x
*e+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e-(-c^2*d
*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^3*b*(c^2*x^2+1)^(1/2)/((c
^2*x^2+1)/c^2/x^2)^(1/2)*x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e
)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(
1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^3*b*(c^2*x
^2+1)^(1/2)/e/((c^2*x^2+1)/c^2/x^2)^(1/2)/x*d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e
)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d-e)/e)^
(1/2)*(c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2))
)+1/4*c*b*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*d-e)/(-c
*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2)
)+1/4*c*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(c^2*d-e)/(-c*x*e
+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))+1/
8*c^3*b/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(
c*x*e+(-c^2*d*e)^(1/2))+1/8*c*b/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(c^2*d-e)/(-c
*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))-1/8*c*b*(c^2*x^2+1)^(1/2)*e
/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c
^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2
+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))-1/8*c*b*(c^2*
x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(
```

```
-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d-e)/e)^(1/2
)*(c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))-1/8
*c*b*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d-e)/e)^(1/
2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(-2*((-(c
^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*
d*e)^(1/2)))-1/8*c*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2
*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))
*ln(-2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-
c*x*e+(-c^2*d*e)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}b\left(\frac{2c^4d^4\log(c) - 2\left(c^4d^2e^2 - 2c^2de^3 + e^4\right)x^4\log(x) + 2d^2e^2\log(c) + d^2e^2 - \left(4d^3e\log(c) + d^3e\right)c^2 + \left(4c^4d^3e\log(c) \cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b*((2*c^4*d^4*log(c) - 2*(c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4*log(x) +
 2*d^2*e^2*log(c) + d^2*e^2 - (4*d^3*e*log(c) + d^3*e)*c^2 + (4*c^4*d^3*e*l
og(c) + 4*d*e^3*log(c) + d*e^3 - (8*d^2*e^2*log(c) + d^2*e^2)*c^2)*x^2 + (c
^4*d^4 - 2*c^2*d^3*e + (c^4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + 2*(c^4*d^3*e - 2*c
^2*d^2*e^2)*x^2)*log(c^2*x^2 + 1) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2 + 2*
(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*x^2)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d
^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^
4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2) + log(e*x^2 + d)/(c^4*d^
3 - 2*c^2*d^2*e + d*e^2) - 8*integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)/(c^2*e^4
*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3)*x^2 + (c
^2*e^4*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3)*x^
2)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d
^2*e^2)
```

Fricas [B] time = 4.94326, size = 2850, normalized size = 17.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e - 2*a*c^
2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d
^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e + e^2)*log((c^2*e*x^2
 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/
(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 -
2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^
2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 4*(b*c^4*d^4 - 2*b*c^
2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^
4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e
 - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + 2*((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^
2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4
 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e
^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*
c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*
b*e^3)*x^4 - 2*b*d^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(c^2*d*e - e^
2)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d - e
)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*
e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e +
b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e -
2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
 - 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e - 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + ((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*
e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5
)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\left(ex^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

3.113 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=205

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bcx\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\sqrt{e}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}}+\frac{bcx\sqrt{-c^2x^2-1}}{8d\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(8*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*(d + e*x^2)) -
 (a + b*ArcCsch[c*x])/(4*e*(d + e*x^2)^2) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2
]])/(4*d^2*e*Sqrt[-(c^2*x^2)]) + (b*c*(3*c^2*d - 2*e)*x*ArcTanh[(Sqrt[e]*Sq
rt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d^2*(c^2*d - e)^(3/2)*Sqrt[e]*Sqrt[-
(c^2*x^2)])
```

Rubi [A] time = 0.204717, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6300, 446, 103, 156, 63, 205, 208}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bcx\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\sqrt{e}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}}+\frac{bcx\sqrt{-c^2x^2-1}}{8d\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(8*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*(d + e*x^2)) -
 (a + b*ArcCsch[c*x])/(4*e*(d + e*x^2)^2) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2
]])/(4*d^2*e*Sqrt[-(c^2*x^2)]) + (b*c*(3*c^2*d - 2*e)*x*ArcTanh[(Sqrt[e]*Sq
rt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d^2*(c^2*d - e)^(3/2)*Sqrt[e]*Sqrt[-
(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
```

```
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^2}\,dx}{4e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\,\text{Subst}\left(\int \frac{c^2d-e-\frac{1}{2}c^2ex}{x\sqrt{-1-c^2x}(d+ex)}\,dx,x,x^2\right)}{8d\left(c^2d-e\right)e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{8d^2e\sqrt{-c^2x^2}}-\frac{(bc\cdots}{\cdots}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{(bx)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{-1-c^2x^2}\right)}{4cd^2e\sqrt{-c^2x^2}}\cdots$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\tan^{-1}\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bc\left(3c^2d-2e\cdots\right.}{8d^2\left(c^2\cdots\right.}$$

Mathematica [C] time = 0.88306, size = 368, normalized size = 1.8

$$\frac{1}{16}\left(-\frac{4a}{e\left(d+ex^2\right)^2}+\frac{b\left(3c^2d-2e\right)\log\left(\frac{16d^2\sqrt{e}\sqrt{e-c^2d}\left(\sqrt{e}+cx\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}-ic\sqrt{d}\right)\right)}{b\left(2e-3c^2d\right)\left(\sqrt{e}x+i\sqrt{d}\right)}\right)}{d^2\sqrt{e}\left(e-c^2d\right)^{3/2}}+\frac{b\left(3c^2d-2e\right)\log\left(-\frac{16id^2\sqrt{e}\sqrt{e-c^2d}\left(\sqrt{e}+cx\left(\cdots\right.\right.}{b\left(3c^2d-2e\right)\left(\cdots\right.}\right.}{d^2\sqrt{e}\left(e-c^2d\right)^{3/2}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d - e)*
(d + e*x^2)) - (4*b*ArcCsch[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSinh[1/(c*x)]
)/(d^2*e) + (b*(3*c^2*d - 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(Sqrt
```

```
[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*
(-3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(-(c^2*d) + e)^(3/
2)) + (b*(3*c^2*d - 2*e)*Log[((-16*I)*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(Sqrt[
e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(3*c
^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*Sqrt[e]*(-(c^2*d) + e)^(3/2)))/
16
```

Maple [B] time = 0.279, size = 1884, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arccsch(c*
x)-1/4*c^3*b*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*d-e)/
(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1
/2))-1/4*c^3*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(c^2*d-e)/(-
c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2
))+3/16*c^3*b*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d-
e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln
(2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e
+(-c^2*d*e)^(1/2)))+3/16*c^3*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2
)/x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d
*e)^(1/2))*ln(2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*
c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+3/16*c^3*b*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1
)/c^2/x^2)^(1/2)*x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2
))/(c*x*e+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e
+(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+3/16*c^3*b*(c^2*x^2+1)^
(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+
(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d-e)/e)^(1/2)*(c^2
*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/4*c*b*
(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d-e)/(-c*x*e+(-c^2
*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))*e^2+1/4*
c*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d-e)/(-c*x*e+(-c
^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))*e-1/8*
c^3*b/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(
c*x*e+(-c^2*d*e)^(1/2))*e-1/8*c*b/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d-e)
/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*e-1/8*c*b*(c^2*x^2+1)^(
1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d^2/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x
*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d-e)/e)^(1/2)*(c
```

```
^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*e^2-1/8
*c*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)
/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*
d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)
^(1/2)))*e-1/8*c*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d^2/(-(c
^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2
))*ln(-2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/
(-c*x*e+(-c^2*d*e)^(1/2)))*e^2-1/8*c*b*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x
*e+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d-e)/e)^(1/2)*(c^2*x^2+1)^(1/2)*e+(-c^2*
d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\left(8\,c^2\int\frac{x}{4\left(c^2e^3x^6+\left(2\,c^2de^2+e^3\right)x^4+d^2e+\left(c^2d^2e+2\,de^2\right)x^2+\left(c^2e^3x^6+\left(2\,c^2de^2+e^3\right)x^4+d^2e+\left(c^2d^2e+2\,de^2\right)x^2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*c^2*integrate(1/4*x/(c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*e
+ (c^2*d^2*e + 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*
e + (c^2*d^2*e + 2*d*e^2)*x^2)*sqrt(c^2*x^2 + 1)), x) + (2*c^2*d - e)*log(e
*x^2 + d)/(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2) - (2*c^4*d^4*log(c) + 2*d^2*e^2
*log(c) - d^2*e^2 - (4*d^3*e*log(c) - d^3*e)*c^2 + (c^2*d^2*e^2 - d*e^3)*x^
2 + (c^4*d^2*e^2*x^4 + 2*c^4*d^3*e*x^2 + c^4*d^4)*log(c^2*x^2 + 1) - 2*((c^
4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*
x^2)*log(x) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2)*log(sqrt(c^2*x^2 + 1) + 1
))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^
2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2))*b - 1/4*a/(e^3
*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Fricas [B] time = 4.78139, size = 2596, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2
*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt
(-c^2*d*e + e^2)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^3*e
+ b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e
- 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x + 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c
^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3
*e + b*d^2*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*((b*
c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c
^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2),
 -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2*
d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(
c^2*d*e - e^2)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
)/(c^2*d - e)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2
- 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*
x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^4*d^4 - 2*b*
c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*
c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - c*x - 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - ((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 +
(b*c^3*d^3*e - b*c*d^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^6*e -
2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(
c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^3, x)
```

3.114 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=657

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csc}}}{\sqrt{e-c^2d}}\right)}{2d^3}$$

```
[Out] -(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(8*d^2*(c^2*d - e)*(e + d/x^2)*x) + (e^2*(a
+ b*ArcCsch[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsch[c*x]))/(d^3*(e
 + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2*b*d^3) - (b*(c^2*d - 2*e)*Sqrt[e]*Ar
cTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d -
e)^(3/2)) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^
2)]*x)])/(d^3*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - (b*PolyLog[2,
 -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^3) -
(b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^3) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e]))])/(2*d^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*d^3)
```

Rubi [A] time = 1.32214, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6304, 5791, 5787, 382, 377, 205, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csc}}}{\sqrt{e-c^2d}}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(8*d^2*(c^2*d - e)*(e + d/x^2)*x) + (e^2*(a
+ b*ArcCsch[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsch[c*x]))/(d^3*(e
 + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2*b*d^3) - (b*(c^2*d - 2*e)*Sqrt[e]*Ar
cTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d -
e)^(3/2)) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^
2)]*x)])/(d^3*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - (b*PolyLog[2,
 -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^3) -
(b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^3) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e]))])/(2*d^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*d^3)
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{e^2x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2ex\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\left(\int \frac{x\left(a\right.}{}\right.}{}$$

$$= \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right.\right.}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right.\right.}{d^2}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\right.}{\sqrt{e}-\sqrt{-d}x}\right.}{2(-d)^{5/2}}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d-}}{c\sqrt{e}\sqrt{1+}}\right.}{d^3\sqrt{c^2d-e}}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2bd^3}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2bd^3}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2bd^3}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{2bd^3}$$

Mathematica [F] time = 63.8584, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3), x]
```

Maple [F] time = 0.492, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a\left(\frac{2\,ex^2+3\,d}{d^2e^2x^4+2\,d^3ex^2+d^4}-\frac{2\,\log\left(ex^2+d\right)}{d^3}+\frac{4\,\log(x)}{d^3}\right)+b\int \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^
3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\mathrm{arcsch}\,(cx) + a}{e^3x^7 + 3\,de^2x^5 + 3\,d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^3*x), x)
```

3.115 $\int \frac{x^4\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1106

result too large to display

```
[Out] -(b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(3/2)*(Sqrt[-d]*Sqr
t[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(3/2)
*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*(S
qrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt
[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e]
 + d/x)^2) - (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (
3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[
1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2*d - e]*e^2) + (b*ArcTanh[(c^2*d -
(Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(
16*Sqrt[d]*(c^2*d - e)^(3/2)*e) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/
x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2
*d - e]*e^2) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^
2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) + (3*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2
*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt
[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
 + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + S
qrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*Pol
yLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(16
*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rubi [A] time = 1.74076, antiderivative size = 1106, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6304, 5706, 5801, 731, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16\left(c^2d-e\right)e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a\right.}{16e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(3/2)*(Sqrt[-d]*Sqr
t[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(3/2)
*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*(S
qrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt
[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e]
 + d/x)^2) - (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (
3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[
1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2*d - e]*e^2) + (b*ArcTanh[(c^2*d -
(Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(
16*Sqrt[d]*(c^2*d - e)^(3/2)*e) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/
x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2
*d - e]*e^2) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^
2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) + (3*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2
*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt
[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
 + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + S
qrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*Pol
yLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(16
*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int\left(-\frac{d^3\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\sinh^{-1}\left(\right.\right.}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}\right.\right.$$

$$= \frac{(3d)\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3\left(a+b\operatorname{csch}^{-1}\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

Mathematica [C] time = 6.19741, size = 2045, normalized size = 1.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*((I*c*Sqrt[e]
*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x))
- ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d
*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*
c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(S
qrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/e^2 - ((I/16)*Sqrt[d]*(((-
I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqr
t[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*
x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*
Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2
*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/e^2 + (5*(-(ArcC
sch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2
*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 +
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*
d) + e]))/Sqrt[d]))/(16*e^2) + (5*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e
*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*
(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*e^2) + (
((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-
2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*Ar
cCsch[c*x])] + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])]))/(Sqrt[d]*e^(5/2)) - (((3*I)/128)*(Pi^2 - (4*I)*Pi*
ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
```

```
t[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*L
og[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*
ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*
Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Lo
g[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*Ar
cCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sq
rt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (
(-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyL
og[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(Sqr
t[d]*e^(5/2)))
```

Maple [F] time = 1.855, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b\operatorname{arccsch}(cx))}{(ex^2 + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)

[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arcsch}(cx) + ax^4}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^4}{\left(ex^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

3.116 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1106

result too large to display

```
[Out] -(b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqrt[e]*(Sqrt[-d]*Sqr
t[e] - d/x)) - (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqrt[e]
*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqr
t[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] -
d/x)) - (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^
2) - (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c
^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2
)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e
])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*Sqrt[
c^2*d - e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c
^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcT
anh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c
^2*x^2)])])/(16*d^(3/2)*Sqrt[c^2*d - e]*e) - ((a + b*ArcCsch[c*x])*Log[1 -
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)
*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsch[c*x])
*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(
-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(16*(-d)^
(3/2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^Ar
cCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*
PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16
*(-d)^(3/2)*e^(3/2))
```

Rubi [A] time = 3.04614, antiderivative size = 1106, normalized size of antiderivative = 1., number of steps used = 63, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6304, 5791, 5706, 5801, 731, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\text{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\text{csch}^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqrt[e]*(Sqrt[-d]*Sqr
t[e] - d/x)) - (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqrt[e]
*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqr
t[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] -
d/x)) - (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^
2) - (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c
^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2
)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e
])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*Sqrt[
c^2*d - e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c
^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcT
anh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c
^2*x^2)])])/(16*d^(3/2)*Sqrt[c^2*d - e]*e) - ((a + b*ArcCsch[c*x])*Log[1 -
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)
*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsch[c*x])
*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(
-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(16*(-d)^
(3/2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^Ar
cCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*
PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16
*(-d)^(3/2)*e^(3/2))
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3}+\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int\left(-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int\left(-\ldots\right.\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} - \frac{3\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} + \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}}{\left(\sqrt{-d}\sqrt{e}-\ldots\right.}\right.}{4e}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\text{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\text{csch}^{-1}}{16de\left(\sqrt{-d}\ldots\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\text{csch}^{-1}}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-\ldots}\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\text{csch}^{-1}}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-\ldots}\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\text{csch}^{-1}}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-\ldots}\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\text{csch}^{-1}}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-\ldots}\right.}$$

$$bc\sqrt{1+\frac{1}{c^2x^2}} \qquad bc\sqrt{1+\frac{1}{c^2x^2}} \qquad + \quad a+b\text{csch}^{-1}$$

Mathematica [C] time = 6.1274, size = 2053, normalized size = 1.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(a*x)/(4*e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-I/16)*((I*c*Sqrt[e]*Sqrt[1 + 1/(c
^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]
/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I
*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] -
 Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqr
t[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*c*Sqrt[e]*S
qrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - Arc
Csch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]
) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c
*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[
d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) - (-(ArcCsch[c*x]/
(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*
Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^
2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))
/Sqrt[d])/(16*d*e) - (-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Ar
cSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d]
+ Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d
] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d])/(16*d*e) + ((I/128)*(Pi^2
- (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*
x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] +
 (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
(4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I
)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*P
olyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])]))/(d^(3/2)*e^(3/2)) - ((I/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCs
ch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqr
t[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*A
```

```
rcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (
I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*Arc
Sin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*
(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin
[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]
 + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(d^(3/2)*e^(3/2)))
```

Maple [F] time = 3.93, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b\,\mathrm{arccsch}\,(cx))}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arcsch}\,(cx) + ax^2}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\left(ex^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

3.117 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1096

result too large to display

```
[Out] -(b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d - e)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d
- e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^(3
/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[-d
]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]
*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d
/x)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d - e]) + (b*e*ArcTanh[(c^
2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)
])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[
e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt
[c^2*d - e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (3*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2
*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt
[e]) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-
d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[
-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]
) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

Rubi [A] time = 3.74264, antiderivative size = 1096, normalized size of antiderivative = 1., number of steps used = 81, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6294, 5791, 5706, 5801, 731, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(c^2d-e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\text{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\text{csch}^{-1}(cx)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d - e)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d
- e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^(3
/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[-d
]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]
*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d
/x)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d - e]) + (b*e*ArcTanh[(c^
2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)
])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[
e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt
[c^2*d - e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (3*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2
*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt
[e]) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-
d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[
-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]
) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

<u>Rule 6294</u>

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

<u>Rule 5791</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(}{\left(e+dx^2\right)^3}\right.}{d^2}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\sin\right.}{4e\left(\sqrt{-d}\right.}\right.\right.}{d^2}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{-de-d^2x^2}\right.}{8d}$$

$$= \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\text{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5\left(a+b\text{csch}^{-1}(c\right.}{16d^2\left(\sqrt{-d}\sqrt{e}+\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right.}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right.}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right.}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right.}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right.}$$

Mathematica [C] time = 6.05857, size = 2038, normalized size = 1.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*((I*c*Sqrt[e]*Sqrt[1 + 1/
(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*
x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) +
(I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d]
 - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*S
qrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*Sqrt[e]*Sqr
t[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCs
ch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e])
+ (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*S
qrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d]
 - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - (3*(-(ArcCsch[c*x]/(I*
Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqr
t[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)]
)*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sq
rt[d]))/(16*d^2) - (3*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(A
rcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[
d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*d^2) + (((3*I)/128)*
(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcC
sch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*
x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
- (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])]
 + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])]))/(d^(5/2)*Sqrt[e]) - (((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x]
 - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcT
an[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) +
```

```
e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]
*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
(16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] +
 Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16
*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*S
qrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(d^(5/2)*Sqrt[e]
))
```

Maple [F] time = 2., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\operatorname{arcsch}(cx) + a}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^3, x)
```

3.118 $\int x^5\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=413

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{7e^3}-\frac{bx\sqrt{-c^2x^2-1}\,(}{}$$

```
[Out] -(b*(23*c^4*d^2 - 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2]
)/(1680*c^5*e^2*Sqrt[-(c^2*x^2)]) - (b*(29*c^2*d + 25*e)*x*Sqrt[-1 - c^2*x^
2]*(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x
^2]*(d + e*x^2)^(5/2))/(42*c*e^2*Sqrt[-(c^2*x^2)]) + (d^2*(d + e*x^2)^(3/2)
*(a + b*ArcCsch[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]
))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^3) + (b*(105*c^6
*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^
2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*c*
d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(105*e^3*Sq
rt[-(c^2*x^2)])
```

Rubi [A] time = 1.4122, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6302, 12, 1615, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{7e^3}-\frac{bx\sqrt{-c^2x^2-1}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```


```
[Out] -(b*(23*c^4*d^2 - 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2]
)/(1680*c^5*e^2*Sqrt[-(c^2*x^2)]) - (b*(29*c^2*d + 25*e)*x*Sqrt[-1 - c^2*x^
2]*(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x
^2]*(d + e*x^2)^(5/2))/(42*c*e^2*Sqrt[-(c^2*x^2)]) + (d^2*(d + e*x^2)^(3/2)
*(a + b*ArcCsch[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]
))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^3) + (b*(105*c^6
*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^
2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*c*
d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(105*e^3*Sq
rt[-(c^2*x^2)])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)dx=\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex\right.}{}$$

$$=\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex\right.}{}$$

$$=\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex\right.}{}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce^2\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}}{}$$

$$=-\frac{b\left(29c^2d+25e\right)x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce^2\sqrt{-c^2x^2}}+\frac{d^2\left(d\right.}{}$$

$$=-\frac{b\left(23c^4d^2-12c^2de-75e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}-\frac{b\left(29c^2d+25e\right)x\sqrt{-1-c^2}}{840c^3e^2\sqrt{-c^2}}$$

$$=-\frac{b\left(23c^4d^2-12c^2de-75e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}-\frac{b\left(29c^2d+25e\right)x\sqrt{-1-c^2}}{840c^3e^2\sqrt{-c^2}}$$

$$=-\frac{b\left(23c^4d^2-12c^2de-75e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}-\frac{b\left(29c^2d+25e\right)x\sqrt{-1-c^2}}{840c^3e^2\sqrt{-c^2}}$$

$$=-\frac{b\left(23c^4d^2-12c^2de-75e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}-\frac{b\left(29c^2d+25e\right)x\sqrt{-1-c^2}}{840c^3e^2\sqrt{-c^2}}$$

$$=-\frac{b\left(23c^4d^2-12c^2de-75e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}-\frac{b\left(29c^2d+25e\right)x\sqrt{-1-c^2}}{840c^3e^2\sqrt{-c^2}}$$

Mathematica [C] time = 0.678312, size = 345, normalized size = 0.84

$$\frac{\sqrt{d+ex^2}\left(16ac^5\left(-4d^2ex^2+8d^3+3de^2x^4+15e^3x^6\right)+bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^4\left(-41d^2+22dex^2+40e^2x^4\right)-2c^2e\left(19d+25ex^2\right)\right.\right.}{1680c^5e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -(b*(128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)
), -(d/(e*x^2))] - (e*(105*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*
Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^
2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(3360*c^5*e^3*x*Sqrt[d + e*x^2])
 + (Sqrt[d + e*x^2]*(16*a*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x
^6) + b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(75*e^2 - 2*c^2*e*(19*d + 25*e*x^2) + c^4
*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d
*e^2*x^4 + 15*e^3*x^6)*ArcCsch[c*x]))/(1680*c^5*e^3)
```

Maple [F] time = 0.457, size = 0, normalized size = 0.

$$\int x^5\left(a+b\,\mathrm{arccsch}\left(cx\right)\right)\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 20.5525, size = 4346, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^
2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e +
 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x
^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e^3*x^6
 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log
((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 48
*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 +
 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 -
 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3)
, 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^
2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e +
 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*
x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c
^2*d*e + e^2)*x^2 + d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*
c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7
*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c
^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2
*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^7*sqrt(-d
)*d^3*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (10
5*b*c^6*d^3 + 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + e^2) + 64*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*
b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a
*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*
b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2
```

```
)))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*(128*b*c^7*sqrt(-d)*d^3*arctan(1/2*(
(c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c
^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (105*b*c^6*d^3 + 35*b
*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^
2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^
4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*
e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*
x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

3.119 $\int x^3\sqrt{d+ex^2}\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=302

$$-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^2\sqrt{-c^2x^2}}-\frac{bx\left(15c^4d^2+10c^2\right.}{120}$$

```
[Out] (b*(c^2*d - 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[-(c^
2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[-(c^2*x^
2)]) - (d*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + ((d + e*x^2)^(5
/2)*(a + b*ArcCsch[c*x]))/(5*e^2) - (b*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*x*
ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(3/2)*
Sqrt[-(c^2*x^2)]) - (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-
1 - c^2*x^2])])/(15*e^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.433657, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6302, 12, 573, 154, 157, 63, 217, 203, 93, 204}

$$-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^2\sqrt{-c^2x^2}}-\frac{bx\left(15c^4d^2+10c^2\right.}{120}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(c^2*d - 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[-(c^
2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[-(c^2*x^
2)]) - (d*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + ((d + e*x^2)^(5
/2)*(a + b*ArcCsch[c*x]))/(5*e^2) - (b*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*x*
ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(3/2)*
Sqrt[-(c^2*x^2)]) - (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-
1 - c^2*x^2])])/(15*e^2*Sqrt[-(c^2*x^2)])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)dx = -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{(bcx)\int\frac{(d+e}{15}}{\sqrt{}}$$

$$=-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{(bcx)\int\frac{(d+e}{}}{15e}$$

$$=-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{(bcx)\,\text{Subst}}{}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+\right.}{5e^2}$$

$$=\frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(\right.}{3}$$

$$=\frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(\right.}{3}$$

$$=\frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(\right.}{3}$$

$$=\frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(\right.}{3}$$

$$=\frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(\right.}{3}$$

Mathematica [A] time = 0.740532, size = 337, normalized size = 1.12

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(-2d^2+dex^2+3e^2x^4\right)+8bc^3\text{csch}^{-1}(cx)\left(-2d^2+dex^2+3e^2x^4\right)+bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(7d+6ex^2\right)-9e\right)\right)}{120c^3e^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(-2*d^2 + d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/(
c^2*x^2)]*x*(-9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^3*(-2*d^2 + d*e*x^2 + 3*e^
2*x^4)*ArcCsch[c*x]))/(120*c^3*e^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-(Sqrt[c^
2]*Sqrt[c^2*d - e]*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d +
 e*x^2))/(c^2*d - e)]*ArcSinh[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt
[c^2*d - e])]) + 16*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c
^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^6*e^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2]
)
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int x^3 (a + b\operatorname{arccsch}(cx)) \sqrt{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.54428, size = 3614, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
+ d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e - 9*b*
e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2
)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c
^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
+ 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x^3 +
 (7*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 +
 d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4
 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*
sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*
c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e
*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e +
e^2)*x^2 + d*e)) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(
e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5
*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e
 - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2
), -1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d
^2 + d*e)*x^2 + d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log
(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4
*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) + e^2) - 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(24*a*c^5*e^
2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e -
9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2),
-1/240*(16*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt
(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2
+ d*e)*x^2 + d^2)) - (15*b*c^4*d^2 + 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arcta
n(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2
+ 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 16*(3*b*c^5*e^
2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a
*c^5*d^2 + (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2
+ 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

3.120 $\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)dx$

Optimal. Leaf size=203

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{bx\left(3c^2d-e\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}}$$

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + ((d + e*x
^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (b*(3*c^2*d - e)*x*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[-(c^2*x^2)])
+ (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e
*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.205643, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6300, 446, 102, 157, 63, 217, 203, 93, 204}

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{bx\left(3c^2d-e\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + ((d + e*x
^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (b*(3*c^2*d - e)*x*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[-(c^2*x^2)])
+ (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e
*Sqrt[-(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{(bcx)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{-1-c^2x^2}}dx}{3e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{(bcx)\,\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{6e\sqrt{-c^2x^2}}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{(bx)\,\text{Subst}\left(\int\frac{-c^2d^2-\frac{1}{2}(\cdots}{x\sqrt{-1-c^2\cdots}}\cdots\right.}{6ce\sqrt{-c^2\cdots}}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{\left(b\left(3c^2d-e\right)x\right)\text{Subst}\left(\cdots\right.}{12c\sqrt{\cdots}}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{\left(b\left(3c^2d-e\right)x\right)\text{Subst}\cdots}{6\cdots}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c\cdots}}\cdots\right.}{3e\sqrt{-c^2x^2}}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{b\left(3c^2d-e\right)x\tan^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.}{6c^2\sqrt{e}\sqrt{-c^2x^2}\cdots}$$

Mathematica [A] time = 0.506957, size = 278, normalized size = 1.37

$$\frac{\sqrt{d+ex^2}\left(2ac\left(d+ex^2\right)+bex\sqrt{\frac{1}{c^2x^2}+1}+2bc\,\text{csch}^{-1}(cx)\left(d+ex^2\right)\right)}{6ce}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(2c^5d^{3/2}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right)\cdots\right.}{6c^4e\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a*c*(d + e*x^2) + 2*b*c*(
d + e*x^2)*ArcCsch[c*x]))/(6*c*e) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-(Sqrt[c^2]
*Sqrt[c^2*d - e]*(3*c^2*d - e)*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d - e)]*
ArcSinh[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e])]) + 2*c^5
*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^
2]]))/(6*c^4*e*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.466, size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{arccsch}\left(cx\right)\right)\sqrt{ex^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(\frac{\left(ex^{2}+d\right)^{\frac{3}{2}}\log\left(\sqrt{c^{2}x^{2}+1}+1\right)}{e}+3\int\frac{\left(c^{2}ex^{3}+c^{2}dx\right)\sqrt{ex^{2}+d}}{3\left(c^{2}ex^{2}+\left(c^{2}ex^{2}+e\right)\sqrt{c^{2}x^{2}+1}+e\right)}\,dx-3\int\frac{\left((3\,e\log\left(c\right)+e)c^{2}x^{3}+\left(c^{2}d+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((e*x^2 + d)^(3/2)*log(sqrt(c^2*x^2 + 1) + 1)/e + 3*integrate(1/3*(c^2*
e*x^3 + c^2*d*x)*sqrt(e*x^2 + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^2
+ 1) + e), x) - 3*integrate(1/3*((3*e*log(c) + e)*c^2*x^3 + (c^2*d + 3*e*lo
g(c))*x + 3*(c^2*e*x^3 + e*x)*log(x))*sqrt(e*x^2 + d)/(c^2*e*x^2 + e), x))*
b + 1/3*(e*x^2 + d)^(3/2)*a/e
```

Fricas [A] time = 5.43913, size = 2984, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e
^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 +
 (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*d^(3/2)*log(
((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)
*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d
^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)
*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (
c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log(
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 + b*c^2*e
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/2
4*(4*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 +
 d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x
^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*
d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(
e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 8*(b*c^3*e*x^2 +
b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*
sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e
)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^
2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan
(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 +
 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2
+ b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + 2*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d
)*sqrt(e*x^2 + d))/(c^3*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

3.121 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi [A] time = 0.0997737, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A] time = 5.3929, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [A] time = 0.452, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x}\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b\operatorname{arcsch}(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi [A] time = 0.10563, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]

[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [A] time = 5.63227, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Maple [A] time = 0.453, size = 0, normalized size = 0.

$$\int \frac{a + b\mathrm{arccsch}\,(cx)}{x^3}\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

3.123 $\int x^2\sqrt{d+ex^2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.100564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 9.07757, size = 0, normalized size = 0.

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.448, size = 0, normalized size = 0.

$$\int x^2 (a + b\,\text{arccsch}\,(cx)) \sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2\,\text{arcsch}\,(cx) + ax^2\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

3.124 $\int \sqrt{d+ex^2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.0368549, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 2.58379, size = 0, normalized size = 0.

$$\int \sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.464, size = 0, normalized size = 0.

$$\int (a + b\text{arccsch}\,(cx))\,\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex^2 + d}(b\,\text{arcsch}\,(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{acsch}\,(cx))\,\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi [A] time = 0.0898209, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 1.70099, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [A] time = 0.449, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^2}\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=389

$$-\frac{bex\left(c^2d-3e\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3dx^3}-\frac{2bc^3x^2\left(c^2d-2e\right)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

```
[Out] (-2*b*c^3*(c^2*d - 2*e)*x^2*Sqrt[d + e*x^2])/(9*d*Sqrt[-(c^2*x^2)]*Sqrt[-1
- c^2*x^2]) - (2*b*c*(c^2*d - 2*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d
*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[-
(c^2*x^2)]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2*b*c^2
*(c^2*d - 2*e)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*
d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
- (b*(c^2*d - 3*e)*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)
])/(9*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*
x^2))])
```

Rubi [A] time = 0.427077, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {264, 6302, 12, 474, 583, 531, 418, 492, 411}

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3dx^3}-\frac{bex\left(c^2d-3e\right)\sqrt{d+ex^2}\,F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{2bc^3x^2\left(c^2d-2e\right)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}-\frac{2bc\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```


```
[Out] (-2*b*c^3*(c^2*d - 2*e)*x^2*Sqrt[d + e*x^2])/(9*d*Sqrt[-(c^2*x^2)]*Sqrt[-1
- c^2*x^2]) - (2*b*c*(c^2*d - 2*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d
*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[-
(c^2*x^2)]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2*b*c^2
*(c^2*d - 2*e)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*
d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
- (b*(c^2*d - 3*e)*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)
])/(9*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*
x^2))])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{3/2}}{3dx^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} + \frac{(bcx)\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\sqrt{-1-c^2x^2}}\,dx}{3d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int \frac{2d\left(c^2d-2e\right)+\left(c^2d-3e\right)\ldots}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}\ldots}}{9d\sqrt{-c^2x^2}}$$

$$= -\frac{2bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\ldots\right.}{3dx^3}$$

$$= -\frac{2bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\ldots\right.}{3dx^3}$$

$$= -\frac{2bc^3\left(c^2d-2e\right)x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\ldots}{9x^2\sqrt{-c\ldots}}$$

$$= -\frac{2bc^3\left(c^2d-2e\right)x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\ldots}{9x^2\sqrt{-c\ldots}}$$

<u>Mathematica</u> [C] time = 0.597136, size = 237, normalized size = 0.61

$$-\frac{\sqrt{d+ex^2}\left(3a\left(d+ex^2\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(2c^2dx^2-d-4ex^2\right)+3b\operatorname{csch}^{-1}(cx)\left(d+ex^2\right)\right)}{9dx^3} - \frac{ibcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}\left(\left(-2c^4\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] -(Sqrt[d + e*x^2]*(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d + 2*c^2*d*x^2 - 4*e*x^2)
 + 3*a*(d + e*x^2) + 3*b*(d + e*x^2)*ArcCsch[c*x]))/(9*d*x^3) - ((I/9)*b*c*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 2*e)*Elliptic
E[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-2*c^4*d^2 + 5*c^2*d*e - 3*e^2)*Ell
ipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sqrt[1 + c^2*x^2]*
Sqrt[d + e*x^2])
```

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^4}\sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsch}\,(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=527

$$\frac{2bex\left(6c^4d^2-4c^2de-15e^2\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(\right.}{5}$$

```
[Out] (b*c^3*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*x^2*Sqrt[d + e*x^2])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)
*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*Sqrt[-(c^2*x^2)]) - (b*c*(12*
c^2*d + e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d*x^2*Sqrt[-(c^2*x^2)])
 + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*d*x^4*Sqrt[-(c^2*x^2)]) -
 ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) + (2*e*(d + e*x^2)^(3/2
)*(a + b*ArcCsch[c*x]))/(15*d^2*x^3) - (b*c^2*(24*c^4*d^2 - 19*c^2*d*e - 31
*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (2*
b*e*(6*c^4*d^2 - 4*c^2*d*e - 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x
], 1 - e/(c^2*d)])/(225*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e
*x^2)/(d*(1 + c^2*x^2))])
```

Rubi [A] time = 0.637992, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {271, 264, 6302, 12, 580, 583, 531, 418, 492, 411}

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5dx^5}+\frac{bc^3x^2\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bc\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c^3*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*x^2*Sqrt[d + e*x^2])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)
*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*Sqrt[-(c^2*x^2)]) - (b*c*(12*
c^2*d + e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d*x^2*Sqrt[-(c^2*x^2)])
 + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*d*x^4*Sqrt[-(c^2*x^2)]) -
 ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) + (2*e*(d + e*x^2)^(3/2
```

```
)*(a + b*ArcCsch[c*x]))/(15*d^2*x^3) - (b*c^2*(24*c^4*d^2 - 19*c^2*d*e - 31
*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (2*
b*e*(6*c^4*d^2 - 4*c^2*d*e - 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x
], 1 - e/(c^2*d)])/(225*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e
*x^2)/(d*(1 + c^2*x^2))])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3} - \frac{(bcx)\int \frac{(d+ex^2\cdots}{15d\cdots}}{\sqrt{\cdots}}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3} - \frac{(bcx)\int \frac{(d+ex^2\cdots}{x\cdots}}{15d^2\cdots}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\cdots\right.}{15d^2x^3}$$

$$= -\frac{bc\left(12c^2d+e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a\cdots\right.}{5d\cdots}$$

$$= \frac{bc\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} - \frac{bc\left(12c^2d+e\right)\sqrt{-1-c^2x^2}\sqrt{d+\cdots}}{225dx^2\sqrt{-c^2x^2}}$$

$$= \frac{bc\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} - \frac{bc\left(12c^2d+e\right)\sqrt{-1-c^2x^2}\sqrt{d+\cdots}}{225dx^2\sqrt{-c^2x^2}}$$

$$= \frac{bc^3\left(24c^4d^2-19c^2de-31e^2\right)x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{-1-c^2x^2}\cdots}{225d^2\sqrt{-c^2x^2}}$$

$$= \frac{bc^3\left(24c^4d^2-19c^2de-31e^2\right)x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{-1-c^2x^2}\cdots}{225d^2\sqrt{-c^2x^2}}$$

Mathematica [C] time = 0.665634, size = 314, normalized size = 0.6

$$\frac{\sqrt{d+ex^2}\left(-15a\left(3d^2+dex^2-2e^2x^4\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(3d^2\left(8c^4x^4-4c^2x^2+3\right)+dex^2\left(8-19c^2x^2\right)-31e^2x^4\right)-15b\text{csch}^{-\cdots}\right.}{225d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2
*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 - 19*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8
*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsch[c*x]))/(225*d^2*x^5
```

```
) + ((I/225)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^4
*d^2 - 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] +
(-24*c^6*d^3 + 31*c^4*d^2*e + 23*c^2*d*e^2 - 30*e^3)*EllipticF[I*ArcSinh[Sq
rt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{arccsch}\,(cx)}{x^6}\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}(b\,\mathrm{arcsch}\,(cx)+a)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

3.128 $\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=384

$$-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{bx\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}$$

```
[Out] -(b*(3*c^4*d^2 + 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])
/(560*c^5*e*Sqrt[-(c^2*x^2)]) + (b*(13*c^2*d - 25*e)*x*Sqrt[-1 - c^2*x^2]*(
d + e*x^2)^(3/2))/(840*c^3*e*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d
 + e*x^2)^(5/2))/(42*c*e*Sqrt[-(c^2*x^2)]) - (d*(d + e*x^2)^(5/2)*(a + b*Ar
cCsch[c*x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^2) - (
b*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*x*ArcTan[(Sqrt[e]*Sqr
t[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[-(c^2*x^2)]) -
 (2*b*c*d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(35
*e^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.51424, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6302, 12, 573, 154, 157, 63, 217, 203, 93, 204}

$$-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{bx\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```


```
[Out] -(b*(3*c^4*d^2 + 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])
/(560*c^5*e*Sqrt[-(c^2*x^2)]) + (b*(13*c^2*d - 25*e)*x*Sqrt[-1 - c^2*x^2]*(
d + e*x^2)^(3/2))/(840*c^3*e*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d
 + e*x^2)^(5/2))/(42*c*e*Sqrt[-(c^2*x^2)]) - (d*(d + e*x^2)^(5/2)*(a + b*Ar
cCsch[c*x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^2) - (
b*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*x*ArcTan[(Sqrt[e]*Sqr
t[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[-(c^2*x^2)]) -
 (2*b*c*d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(35
*e^2*Sqrt[-(c^2*x^2)])
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\operatorname{csch}^{-1}(cx)\right) dx = -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} - \frac{(bcx)\int \ldots}{\ldots}$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} - \frac{(bcx)\int \ldots}{3\ldots}$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} - \frac{(bcx)\,\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a \ldots\right.}{7\ldots}$$

$$= \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d\left(d+\ldots\right.}{\ldots}$$

$$= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2\ldots}}{840c^3e\sqrt{-c^2x\ldots}}$$

$$= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2\ldots}}{840c^3e\sqrt{-c^2x\ldots}}$$

$$= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2\ldots}}{840c^3e\sqrt{-c^2x\ldots}}$$

$$= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2\ldots}}{840c^3e\sqrt{-c^2x\ldots}}$$

$$= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2\ldots}}{840c^3e\sqrt{-c^2x\ldots}}$$

Mathematica [C] time = 0.734967, size = 318, normalized size = 0.83

$$\frac{\sqrt{d+ex^2}\left(-48ac^5\left(2d-5ex^2\right)\left(d+ex^2\right)^2 + bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^4\left(57d^2+106dex^2+40e^2x^4\right)-2c^2e\left(82d+25ex^2\right)+75e^2\right)-4\ldots\right.}{1680c^5e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -(b*(-32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)
), -(d/(e*x^2))] + (e*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*S
qrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2
*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(1120*c^5*e^2*x*Sqrt[d + e*x^2])
+ (Sqrt[d + e*x^2]*(-48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*Sqrt[1 +
1/(c^2*x^2)]*x*(75*e^2 - 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*
x^2 + 40*e^2*x^4)) - 48*b*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/
(1680*c^5*e^2)
```

Maple [F] time = 0.459, size = 0, normalized size = 0.

$$\int x^{3} \left(e x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccsch}\left(c x \right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 19.9241, size = 4343, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*d^(7/2)*log((((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e -
 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x
^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e^3*x^6
 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 384*
a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2
*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 +
75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2),
 1/3360*(48*b*c^7*d^(7/2)*log((((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e -
 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*
x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c
^2*d*e + e^2)*x^2 + d*e)) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7
*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d
^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*
e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt((c^2*x
^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2), -1/6720*(192*b*c^7*sqrt(-d)
*d^3*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - 3*(3
5*b*c^6*d^3 + 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + e^2) - 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*
c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 - 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c
^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*
c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^7*e^2), -1/3360*(96*b*c^7*sqrt(-d)*d^3*arctan(1/2*((
c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^
2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - 3*(35*b*c^6*d^3 + 35*b
*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2
*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4
```

```
+ 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2
 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcsch}\,(cx)+a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^3, x)
```

3.129 $\int x\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)dx$

Optimal. Leaf size=270

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5e}+\frac{bx\left(15c^4d^2-10c^2de+3e^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}}+\frac{bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{5e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2\cdots}}{2\cdots}$$

```
[Out] (b*(7*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[-(c^2
*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[-(c^2*x^2)]
) + ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (b*(15*c^4*d^2 - 10*c^
2*d*e + 3*e^2)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/
(40*c^4*Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(5*e*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.281469, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6300, 446, 102, 154, 157, 63, 217, 203, 93, 204}

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5e}+\frac{bx\left(15c^4d^2-10c^2de+3e^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}}+\frac{bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{5e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2\cdots}}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(7*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[-(c^2
*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[-(c^2*x^2)]
) + ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (b*(15*c^4*d^2 - 10*c^
2*d*e + 3*e^2)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/
(40*c^4*Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(5*e*Sqrt[-(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
```

```
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bcx)\int \frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{-1-c^2x^2}}dx}{5e\sqrt{-c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bcx)\,\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx, x, x^2\right)}{10e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{(bx)\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{b\left(7c^2d-3e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5}}{}$$

$$= \frac{b\left(7c^2d-3e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5}}{}$$

$$= \frac{b\left(7c^2d-3e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5}}{}$$

$$= \frac{b\left(7c^2d-3e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5}}{}$$

$$= \frac{b\left(7c^2d-3e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5}}{}$$

Mathematica [A] time = 0.727555, size = 314, normalized size = 1.16

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(d+ex^2\right)^2 + bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(9d+2ex^2\right)-3e\right) + 8bc^3\text{csch}^{-1}(cx)\left(d+ex^2\right)^2\right)}{40c^3e} - \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(\sqrt{c^2}\sqrt{e}\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(-3*e
 + c^2*(9*d + 2*e*x^2)) + 8*b*c^3*(d + e*x^2)^2*ArcCsch[c*x]))/(40*c^3*e) -
```

```
(b*Sqrt[1 + 1/(c^2*x^2)]*x*(Sqrt[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*(-15*c^4*d^2
 + 10*c^2*d*e - 3*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d - e)]*ArcSinh[(c*Sqrt[
e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e])] + 8*c^7*d^(5/2)*Sqrt[-d
- e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^2]]))/(40*c^6*e*S
qrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.447, size = 0, normalized size = 0.

$$\int x\left(ex^{2}+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arccsch}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ex^{2}+d\right)^{\frac{5}{2}}a}{5\,e}+\frac{1}{5}\left(\frac{\left(e^{2}x^{4}+2\,dex^{2}+d^{2}\right)\sqrt{ex^{2}+d}\log\left(\sqrt{c^{2}x^{2}+1}+1\right)}{e}+5\int\frac{\left(c^{2}e^{2}x^{5}+2\,c^{2}dex^{3}+c^{2}d^{2}x\right)\sqrt{ex^{2}+d}}{5\left(c^{2}ex^{2}+\left(c^{2}ex^{2}+e\right)\sqrt{c^{2}x^{2}+1}+e\right)}\,dx-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d
)*log(sqrt(c^2*x^2 + 1) + 1)/e + 5*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*e*x
^3 + c^2*d^2*x)*sqrt(e*x^2 + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^2 +
 1) + e), x) - 5*integrate(1/5*((5*e^2*log(c) + e^2)*c^2*x^5 + ((5*d*e*log(
c) + 2*d*e)*c^2 + 5*e^2*log(c))*x^3 + (c^2*d^2 + 5*d*e*log(c))*x + 5*(c^2*e
^2*x^5 + (c^2*d*e + e^2)*x^3 + d*e*x)*log(x))*sqrt(e*x^2 + d)/(c^2*e*x^2 +
e), x))*b
```

Fricas [A] time = 10.5, size = 3579, normalized size = 13.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
 d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e
^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)
*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*
d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4
*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(
3*b*c^4*d*e - b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))
/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c
^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e
 + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 +
d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^
2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d
)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^5*e^2*x^4 +
 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e - b*c^2
*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), 1/160*(16
*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d
)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2
 + d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^
4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4
*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2)
 + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5
*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x)*
sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), 1/80*(8*b*c^5*sqrt
(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*
sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) -
(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2
 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*
x^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*
x^2 + d))/(c^5*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\operatorname{arcsch}(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x, x)
```

3.130 $\int \frac{(d+ex^2)^{3/2}(a+b\,\mathbf{csch}^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(d+ex^2)^{3/2}(a+b\,\text{csch}^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi [A] time = 0.117036, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\,\text{csch}^{-1}(cx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\,\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\,\text{csch}^{-1}(cx))}{x}\,dx$$

Mathematica [A] time = 6.29387, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}(a+b\,\text{csch}^{-1}(cx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [A] time = 0.449, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x}\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\operatorname{arcsch}(cx)\right)\sqrt{ex^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x,x)

[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcsch}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x, x)
```

3.131 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi [A] time = 0.122784, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [A] time = 5.66067, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Maple [A] time = 0.447, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^3} \left(ex^2 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\mathrm{arcsch}\,(cx)\right)\sqrt{ex^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^3, x)
```

3.132 $\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(x^2 \left(d+ex^2\right)^{3/2} \left(a+b\,\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.119695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]

[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\,\text{csch}^{-1}(cx)\right)\,dx=\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 9.29328, size = 0, normalized size = 0.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]

[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.448, size = 0, normalized size = 0.

$$\int x^{2} \left(e x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccsch}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a e x^{4} + a d x^{2} + \left(b e x^{4} + b d x^{2}\right) \mathrm{arcsch}\left(c x\right)\right) \sqrt{e x^{2} + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arccsch(c*x))*sqrt(e*x^2
+ d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcsch}\,(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^2, x)
```

3.133 $\int \left(d+ex^2\right)^{3/2} \left(a+b\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.0438619, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]

[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx=\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Mathematica [A] time = 3.48192, size = 0, normalized size = 0.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]

[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.459, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccsch}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(aex^{2}+ad+\left(bex^{2}+bd\right)\mathrm{arcsch}\left(cx\right)\right)\sqrt{ex^{2}+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

3.134 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi [A] time = 0.105802, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 5.53719, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]

Maple [A] time = 0.491, size = 0, normalized size = 0.

$$\int \frac{a + b\text{arccsch}\,(cx)}{x^2} \left(ex^2 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)

[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right) \text{arcsch}\,(cx)\right)\sqrt{ex^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^2
, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arcsch}\left(cx\right)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^2, x)
```

3.135 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4},x\right)$$

```
[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Rubi [A] time = 0.107959, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$$

Mathematica [A] time = 15.4703, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Maple [A] time = 0.458, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x^4}\left(ex^2 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\operatorname{arcsch}(cx)\right)\sqrt{ex^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcsch}\left(cx\right)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^4, x)
```

3.136 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=492

$$\frac{bex\left(4c^4d^2-11c^2de+15e^2\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}+\frac{bc^3x^2\left(8c^4d^2-2\right.}{75d\sqrt{-\right.}}$$

```
[Out] (b*c^3*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*x^2*Sqrt[d + e*x^2])/(75*d*Sqrt[-(
c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*Sqrt
[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*Sqrt[-(c^2*x^2)]) - (4*b*c*(c^2*d - 2
*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*x^2*Sqrt[-(c^2*x^2)]) + (b*c*Sq
rt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^4*Sqrt[-(c^2*x^2)]) - ((d + e*x^2
)^(5/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) - (b*c^2*(8*c^4*d^2 - 23*c^2*d*e +
23*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(75*d*Sqrt
[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (b*e
*(4*c^4*d^2 - 11*c^2*d*e + 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x],
 1 - e/(c^2*d)])/(75*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^
2)/(d*(1 + c^2*x^2))])
```

Rubi [A] time = 0.575472, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {264, 6302, 12, 474, 580, 583, 531, 418, 492, 411}

$$-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}+\frac{bc^3x^2\left(8c^4d^2-23c^2de+23e^2\right)\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bc\sqrt{-c^2x^2-1}\left(8c^4d^2-23c^2de+23e^2\right)\sqrt{d}}{75d\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```


```
[Out] (b*c^3*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*x^2*Sqrt[d + e*x^2])/(75*d*Sqrt[-(
c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*Sqrt
[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*Sqrt[-(c^2*x^2)]) - (4*b*c*(c^2*d - 2
*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*x^2*Sqrt[-(c^2*x^2)]) + (b*c*Sq
rt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^4*Sqrt[-(c^2*x^2)]) - ((d + e*x^2
)^(5/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) - (b*c^2*(8*c^4*d^2 - 23*c^2*d*e +
23*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(75*d*Sqrt
```

```
[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]]) + (b*e
*(4*c^4*d^2 - 11*c^2*d*e + 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x],
 1 - e/(c^2*d)])/(75*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^
2)/(d*(1 + c^2*x^2))])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
```

```
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} - \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{5/2}}{5dx^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int \frac{\left(d+ex^2\right)^{5/2}}{x^6\sqrt{-1-c^2x^2}}\,dx}{5d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(4d\left(c^2\right.\right.}{x^4\sqrt{}}}{25d\sqrt{}}$$

$$= -\frac{4bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{5/2}}{}$$

$$= \frac{bc\left(8c^4d^2-23c^2de+23e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{4bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d}}{75x^2\sqrt{-c^2x^2}}$$

$$= \frac{bc\left(8c^4d^2-23c^2de+23e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{4bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d}}{75x^2\sqrt{-c^2x^2}}$$

$$= \frac{bc^3\left(8c^4d^2-23c^2de+23e^2\right)x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\left(8c^4d^2-23c^2de+23e^2\right)\sqrt{-1-c^2x^2}}{75d\sqrt{-c^2x^2}}$$

$$= \frac{bc^3\left(8c^4d^2-23c^2de+23e^2\right)x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\left(8c^4d^2-23c^2de+23e^2\right)\sqrt{-1-c^2x^2}}{75d\sqrt{-c^2x^2}}$$

Mathematica [C] time = 0.682093, size = 291, normalized size = 0.59

$$\frac{\sqrt{d+ex^2}\left(-15a\left(d+ex^2\right)^2 + bcx\sqrt{\frac{1}{c^2x^2}+1}\left(d^2\left(8c^4x^4-4c^2x^2+3\right)+dex^2\left(11-23c^2x^2\right)+23e^2x^4\right) - 15b\operatorname{csch}^{-1}(cx)\left(d+\right.\right.}{75dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(23*e^2
*x^4 + d*e*x^2*(11 - 23*c^2*x^2) + d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*
(d + e*x^2)^2*ArcCsch[c*x]))/(75*d*x^5) + ((I/75)*b*c*Sqrt[1 + 1/(c^2*x^2)]
```

```
*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*EllipticE[I
*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-8*c^6*d^3 + 27*c^4*d^2*e - 34*c^2*d*e
^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sq
rt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{arccsch}\,(cx)}{x^6}\left(ex^2+d\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\mathrm{arcsch}\,(cx)\right)\sqrt{ex^2+d}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^6
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcsch}\,(cx)+a)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^6, x)
```

3.137 $\int \frac{(d+ex^2)^{3/2}(a+b\,\mathbf{csch}^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=643

$$-\frac{bex\left(-249c^4d^2e+120c^6d^3+71c^2de^2+210e^3\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(c\right.}{35d^2x^5}$$

```
[Out] -(b*c^3*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x^2*Sqrt[d
+ e*x^2])/(3675*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) - (b*c*(240*c^6*d^
3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*
x^2])/(3675*d^2*Sqrt[-(c^2*x^2)]) + (b*c*(120*c^4*d^2 - 159*c^2*d*e - 37*e^
2)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[-(c^2*x^2)]) - (b*c
*(30*c^2*d - 11*e)*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^4*Sqrt[-
(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*Sqrt[-(c
^2*x^2)]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(7*d*x^7) + (2*e*(d +
e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 - 528
*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(3675*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d +
 e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(120*c^6*d^3 - 249*c^4*d^2*e + 71*c^2*d*
e^2 + 210*e^3)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(36
75*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2
))])
```

Rubi [A] time = 0.866062, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {271, 264, 6302, 12, 580, 583, 531, 418, 492, 411}

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}-\frac{bc^3x^2\left(-528c^4d^2e+240c^6d^3+193c^2de^2+247e^3\right)}{3675d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] -(b*c^3*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x^2*Sqrt[d
+ e*x^2])/(3675*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) - (b*c*(240*c^6*d^
3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*
```

```
x^2])/(3675*d^2*Sqrt[-(c^2*x^2)]) + (b*c*(120*c^4*d^2 - 159*c^2*d*e - 37*e^
2)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[-(c^2*x^2)]) - (b*c
*(30*c^2*d - 11*e)*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^4*Sqrt[-
(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*Sqrt[-(c
^2*x^2)]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(7*d*x^7) + (2*e*(d +
e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 - 528
*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(3675*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d +
 e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(120*c^6*d^3 - 249*c^4*d^2*e + 71*c^2*d*
e^2 + 210*e^3)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(36
75*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2
))])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{(bcx)\int \frac{(d+}{3}}{}$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{(bcx)\int \frac{(d+}{}}{35}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+\right.}{35d^2x}$$

$$= -\frac{bc\left(30c^2d-11e\right)\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{-c^2x^2}}-\frac{\left(d+ex\right.}{}$$

$$= \frac{bc\left(120c^4d^2-159c^2de-37e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}}-\frac{bc\left(30c^2d-11e\right)\sqrt{-1-c^2x}}{1225dx^4\sqrt{-c^2x}}$$

$$= -\frac{bc\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}}+\frac{bc\left(120c^4d^2\right.}{}$$

$$= -\frac{bc\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}}+\frac{bc\left(120c^4d^2\right.}{}$$

$$= -\frac{bc^3\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{bc\left(240c^6d^3-528c\right.}{}$$

$$= -\frac{bc^3\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{bc\left(240c^6d^3-528c\right.}{}$$

Mathematica [C] time = 0.823573, size = 372, normalized size = 0.58

$$-\frac{\sqrt{d+ex^2}\left(105a\left(5d-2ex^2\right)\left(d+ex^2\right)^2+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(-3d^2ex^2\left(176c^4x^4-83c^2x^2+61\right)+15d^3\left(16c^6x^6-8c^4x^4+6c^2x\right.\right.\right.}{3675d^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] -(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1 + 1/(c^
2*x^2)]*x*(247*e^3*x^6 + d*e^2*x^4*(-71 + 193*c^2*x^2) - 3*d^2*e*x^2*(61 -
83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6
)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/(3675*d^2*x^7) - ((
I/3675)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d^3
 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*EllipticE[I*ArcSinh[Sqrt[c^2]*x
], e/(c^2*d)] - 2*(120*c^8*d^4 - 324*c^6*d^3*e + 221*c^4*d^2*e^2 + 88*c^2*d
*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d
^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x^8}\left(ex^2 + d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\operatorname{arcsch}(cx)\right)\sqrt{ex^2 + d}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^8
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcsch}\left(cx\right)+a\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^8, x)
```

3.138 $\int \frac{x^5\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=329

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}+\frac{8bcd^{5/2}x\tan^{-1}\left(\right.}{15e^3\sqrt{-c}}$$

```
[Out] -(b*(19*c^2*d + 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e^2*Sqr
t[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[
-(c^2*x^2)]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^3 - (2*d*(d + e
*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCs
ch[c*x]))/(5*e^3) + (b*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*x*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqrt[-(c^2*x^2)]
) + (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/
(15*e^3*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 1.17938, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6302, 12, 1615, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}+\frac{8bcd^{5/2}x\tan^{-1}\left(\right.}{15e^3\sqrt{-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -(b*(19*c^2*d + 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e^2*Sqr
t[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[
-(c^2*x^2)]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^3 - (2*d*(d + e
*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCs
ch[c*x]))/(5*e^3) + (b*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*x*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqrt[-(c^2*x^2)]
) + (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/
(15*e^3*Sqrt[-(c^2*x^2)])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx=\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csc}\right.}{5e^3}$$

$$=\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csc}\right.}{5e^3}$$

$$=\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csc}\right.}{5e^3}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}\right.}{3e^3}$$

$$=-\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{cs}\right.}{e^3}$$

$$=-\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{cs}\right.}{e^3}$$

$$=-\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{cs}\right.}{e^3}$$

$$=-\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{cs}\right.}{e^3}$$

$$=-\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{cs}\right.}{e^3}$$

Mathematica [A] time = 0.774949, size = 339, normalized size = 1.03

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(8d^2-4dex^2+3e^2x^4\right)+8bc^3\text{csch}^{-1}(cx)\left(8d^2-4dex^2+3e^2x^4\right)+bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2\left(6ex^2-13d\right)-9e\right)\right)}{120c^3e^3}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/
(c^2*x^2)]*x*(-9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^3*(8*d^2 - 4*d*e*x^2 +
3*e^2*x^4)*ArcCsch[c*x]))/(120*c^3*e^3) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-(Sqr
t[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*Sqrt[(c^2*
(d + e*x^2))/(c^2*d - e)]*ArcSinh[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*
Sqrt[c^2*d - e])]) + 64*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1
 + c^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^6*e^3*Sqrt[1 + c^2*x^2]*Sqrt[d + e*
x^2])
```

Maple [F] time = 0.452, size = 0, normalized size = 0.

$$\int x^{5} \left(a + b \operatorname{arccsch}\left(c x\right)\right) \frac{1}{\sqrt{e x^{2} + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.0586, size = 3636, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
+ d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (45*b*c^4*d^2 + 10*b*c^2*d*e + 9*b*
e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2
)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b
*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^
3 - (13*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2
)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2
+ d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4*d^2 +
10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*s
qrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d
*e + e^2)*x^2 + d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2
)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2
4*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^3 - (13*
b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))
/(c^5*e^3), 1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2
*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 + d*e)*x^2 + d^2)) + (45*b*c^4*d^2 + 10*b*c^2*d*e + 9*b*e^2)*sq
rt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 +
 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^
2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(
24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^3 - (13
*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)
)/(c^5*e^3), 1/240*(64*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2
*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 + d*e)*x^2 + d^2)) - (45*b*c^4*d^2 + 10*b*c^2*d*e + 9*b*e^2)*sq
rt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sq
rt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 16
*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5
*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^3 - (13*b*c^4*d*e + 9*b*c^2*e^2)*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^5}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

3.139 $\int \frac{x^3\left(a+b\textbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=229

$$-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^2\sqrt{-c^2x^2}}-\frac{bx\left(3c^2d+e\right)\tan^{-1}\left(\sqrt{\right.}}{6c^2e^{3/2}\sqrt{-c^2x^2}}$$

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[-(c^2*x^2)]) - (d*Sqrt
[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c
*x]))/(3*e^2) - (b*(3*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*S
qrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[-(c^2*x^2)]) - (2*b*c*d^(3/2)*x*ArcTa
n[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.318005, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6302, 12, 573, 154, 157, 63, 217, 203, 93, 204}

$$-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^2\sqrt{-c^2x^2}}-\frac{bx\left(3c^2d+e\right)\tan^{-1}\left(\sqrt{\right.}}{6c^2e^{3/2}\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[-(c^2*x^2)]) - (d*Sqrt
[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c
*x]))/(3*e^2) - (b*(3*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*S
qrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[-(c^2*x^2)]) - (2*b*c*d^(3/2)*x*ArcTa
n[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^2*Sqrt[-(c^2*x^2)])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\int \frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{3e^2x\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}}$$

$$= -\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\int \frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}}}{3e^2\sqrt{-c^2x^2}}$$

$$= -\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\,\text{Subst}\left(\int \frac{(-2d+ex}{x\sqrt{-1}}\right.}{6e^2\sqrt{-c^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

Mathematica [A] time = 0.542568, size = 280, normalized size = 1.22

$$\frac{\sqrt{d+ex^2}\left(-4acd+2acex^2+bex\sqrt{\frac{1}{c^2x^2}+1}+2bc\text{csch}^{-1}(cx)\left(ex^2-2d\right)\right)}{6ce^2} + \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(4c^5d^{3/2}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2}}{\sqrt{-d-}}\right.\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(-4*a*c*d + b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a*c*e*x^2 + 2*
b*c*(-2*d + e*x^2)*ArcCsch[c*x]))/(6*c*e^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-
```

```
(Sqrt[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*(3*c^2*d + e)*Sqrt[(c^2*(d + e*x^2))/(c^
2*d - e)]*ArcSinh[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e])
]) + 4*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt
[-d - e*x^2]]))/(6*c^4*e^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.452, size = 0, normalized size = 0.

$$\int x^3 (a + b\operatorname{arccsch}(cx)) \frac{1}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.88498, size = 3011, normalized size = 13.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e
```

```
^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 +
 (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*d^(3/2
)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d
+ c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*
d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x
^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 +
 d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 +
 b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3
*e^2), -1/24*(8*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d
^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*
e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) - 8*(b*c
^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) + 1)/(c*x)) - 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
- 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(4*b*c^3*sqrt(-d)*d*arctan(1
/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1
)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d + b*e)
*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)
*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) -
 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 1)/(c*x)) - 2*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcsch}\,(cx) + a)x^3}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

3.140 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}}+\frac{bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e\sqrt{-c^2x^2}}$$

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c
^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[d]*x*
ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.153675, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6300, 446, 105, 63, 217, 203, 93, 204}

$$\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}}+\frac{bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c
^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[d]*x*
ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e*Sqrt[-(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}}\,dx}{e\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{2e\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2\sqrt{-c^2x^2}} - \frac{(bcdx)\,\text{Subst}\left(\int \cdots\right.}{2e\sqrt{\cdots}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{(bx)\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{c\sqrt{-c^2x^2}} - \frac{(bcdx)\,\text{Subst}\cdots}{\cdots}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}} + \frac{(bx)\,\text{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1\cdots}}{\sqrt{d\cdots}}\right.}{c\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.238083, size = 223, normalized size = 1.65

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(c^3\sqrt{d}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right) - \sqrt{c^2}\sqrt{e}\sqrt{c^2d-e}\sqrt{\frac{c^2(d+ex^2)}{c^2d-e}}\sinh^{-1}\left(\frac{c\sqrt{e}\cdots}{\sqrt{c\cdots}}\right.\right.}{c^2e\sqrt{c^2x^2+1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-(Sq
rt[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d - e)]*ArcSinh
[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e])]) + c^3*Sqrt[d]*
Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^2]]))/(c^
2*e*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.46, size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{arccsch}\left(cx\right)\right)\frac{1}{\sqrt{e x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\frac{\sqrt{e x^{2}+d}\log\left(\sqrt{c^{2}x^{2}+1}+1\right)}{e}+\int\frac{c^{2}e x^{3}+c^{2}dx}{\left(c^{2}e x^{2}+e\right)\sqrt{c^{2}x^{2}+1}\sqrt{e x^{2}+d}+\left(c^{2}e x^{2}+e\right)\sqrt{e x^{2}+d}}\,dx-\int\frac{(e\log(c)+e)c^{2}x^{3}+(}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(e*x^2 + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate((c^2*e*x^3 + c^
2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2*e*x^2 + e)
*sqrt(e*x^2 + d)), x) - integrate(((e*log(c) + e)*c^2*x^3 + (c^2*d + e*log(
c))*x + (c^2*e*x^3 + e*x)*log(x))/((c^2*e*x^2 + e)*sqrt(e*x^2 + d)), x)) +
sqrt(e*x^2 + d)*a/e
```

Fricas [B] time = 3.93202, size = 2398, normalized size = 17.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*
x)) + b*c*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*
x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*log(8*c^
```

```
4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^
3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*
(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(
d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - 2*
b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e
)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)))
/(c*e), 1/4*(2*b*c*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e
*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 +
d*e)*x^2 + d^2)) + 4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) + 4*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c
^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2))/(c*e
), 1/2*(b*c*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 +
d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^
2 + d^2)) + 2*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*c - b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c
^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^
2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)))/(c*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{acsch}\left(cx\right)\right)}{\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcsch}\left(cx\right)+a)x}{\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

3.141 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi [A] time = 0.0958852, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]

[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.67482, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [A] time = 0.454, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x}\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

3.142 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi [A] time = 0.10777, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]

[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 22.7983, size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [A] time = 0.455, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x^3} \frac{1}{\sqrt{ex^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{ex^5 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d}x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

3.143 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0915653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]

[Out] Defer[Int][(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 6.25353, size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.451, size = 0, normalized size = 0.

$$\int x^2 (a + b\operatorname{arccsch}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arcsch}(cx) + ax^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

3.144 $\int \frac{a+b\textbf{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0312072, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.03941, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.482, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccsch}(cx)) \frac{1}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx) + a}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.145 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=294

$$\frac{bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{dx}+\frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bc\sqrt{-c^2x^2-1}\sqrt{d+}}{d\sqrt{-c^2x^2}}$$

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*
Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]
*(a + b*ArcCsch[c*x]))/(d*x) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)
/(d*(1 + c^2*x^2))]) + (b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/
(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 +
 c^2*x^2))])
```

Rubi [A] time = 0.250901, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {264, 6302, 12, 475, 21, 422, 418, 492, 411}

$$-\frac{\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{dx}+\frac{bex\sqrt{d+ex^2}F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*
Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]
*(a + b*ArcCsch[c*x]))/(d*x) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)
/(d*(1 + c^2*x^2))]) + (b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/
(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 +
 c^2*x^2))])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}+\frac{(bcx)\int \frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}+\frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x^2\sqrt{-1-c^2x^2}}\,dx}{d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}-\frac{(bcx)\int \frac{-e-c^2ex^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}-\frac{(bcex)\int \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}+\frac{(bcex)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}}+\frac{\left(bc^3ex\right)}{}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}+\frac{bex\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-}}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx}-\frac{bc^2x\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-}}$$

Mathematica [A] time = 0.1997, size = 139, normalized size = 0.47

$$\frac{\sqrt{d+ex^2}\left(-a+bcx\sqrt{\frac{1}{c^2x^2}+1}-b\operatorname{csch}^{-1}(cx)\right)}{dx}-\frac{bcex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{e}{d}}x\right)|\frac{c^2d}{e}\right)}{d\sqrt{c^2x^2+1}\sqrt{-\frac{e}{d}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x - b*ArcCsch[c*x]))/(d*x)
 - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt
[-(e/d)]*x], (c^2*d)/e])/(d*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.455, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x^2}\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{ex^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcsch}(cx)+a}{\sqrt{ex^2+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

3.146 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=425

$$-\frac{bex\left(c^2d+6e\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{2e\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3dx^3}-$$

```
[Out] -(b*c^3*(2*c^2*d + 5*e)*x^2*Sqrt[d + e*x^2])/(9*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-
1 - c^2*x^2]) - (b*c*(2*c^2*d + 5*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*d^2*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*
Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2*e
*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d + 5*e)*x
*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^2*Sqrt[-(c^2*x
^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(c^2*d
+ 6*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^3*Sqrt
[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rubi [A] time = 0.486439, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {271, 264, 6302, 12, 580, 583, 531, 418, 492, 411}

$$\frac{2e\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3dx^3}-\frac{bc^3x^2\left(2c^2d+5e\right)\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}-\frac{bc\sqrt{-c^2x^2-1}\left(2c^2d+5e\right)}{9d^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -(b*c^3*(2*c^2*d + 5*e)*x^2*Sqrt[d + e*x^2])/(9*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-
1 - c^2*x^2]) - (b*c*(2*c^2*d + 5*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*d^2*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*
Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2*e
*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d + 5*e)*x
*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^2*Sqrt[-(c^2*x
^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(c^2*d
+ 6*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^3*Sqrt
[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{(bcx)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{3d^2x^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{(bcx)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{x^4\sqrt{-1-c^2x^2}}\,dx}{3d^2\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}+\frac{(bc}{}$$

$$= -\frac{bc\left(2c^2d+5e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$$= -\frac{bc\left(2c^2d+5e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$$= -\frac{bc^3\left(2c^2d+5e\right)x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{bc\left(2c^2d+5e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}$$

$$= -\frac{bc^3\left(2c^2d+5e\right)x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{bc\left(2c^2d+5e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}$$

Mathematica [C] time = 0.626413, size = 239, normalized size = 0.56

$$-\frac{\sqrt{d+ex^2}\left(3a\left(d-2ex^2\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(2c^2dx^2-d+5ex^2\right)+3b\text{csch}^{-1}(cx)\left(d-2ex^2\right)\right)}{9d^2x^3}-\frac{ibcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}\left(c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -(Sqrt[d + e*x^2]*(3*a*(d - 2*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d + 2*
c^2*d*x^2 + 5*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsch[c*x]))/(9*d^2*x^3) - ((I/9
)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d + 5*e)*El
lipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] - 2*(c^4*d^2 + 2*c^2*d*e - 3*e^2
)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 + c^
2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.491, size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{arccsch}\,(cx)}{x^4}\,\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arcsch}\,(cx)+a)}{ex^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^6 + d*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d}x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

3.147 $\int \frac{x^5\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{d^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^3}-\frac{8bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^3\sqrt{-c^2x^2}}$$

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[-(c^2*x^2)]) - (d^2*
(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*A
rcCsch[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (b*(
9*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6
*c^2*e^(5/2)*Sqrt[-(c^2*x^2)]) - (8*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(S
qrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 1.13459, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 6302, 12, 1615, 157, 63, 217, 203, 93, 204}

$$-\frac{d^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^3}-\frac{8bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```


```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[-(c^2*x^2)]) - (d^2*
(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*A
rcCsch[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (b*(
9*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6
*c^2*e^(5/2)*Sqrt[-(c^2*x^2)]) - (8*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(S
qrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
```

```
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{(d+e}{}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{(d+e}{}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{(d+e}{}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{(d+e}{}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{(d+e}{}$$

Mathematica [A] time = 0.633867, size = 311, normalized size = 1.21

$$\frac{-2ac\left(8d^2+4dex^2-e^2x^4\right)+bex\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)-2bc\text{csch}^{-1}(cx)\left(8d^2+4dex^2-e^2x^4\right)}{6ce^3\sqrt{d+ex^2}} + \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(16c^5d^{3/2}\sqrt{-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*(8*d^2 + 4*d*e*x^2 - e^2*x
^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsch[c*x])/(6*c*e^3*Sqrt[d + e
```

```
*x^2]) + (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-(Sqrt[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*(9
*c^2*d + e)*Sqrt[(c^2*(d + e*x^2))/(c^2*d - e)]*ArcSinh[(c*Sqrt[e]*Sqrt[1 +
 c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e])]) + 16*c^5*d^(3/2)*Sqrt[-d - e*x^2]*
ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^4*e^3*Sqrt[1 +
c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.472, size = 0, normalized size = 0.

$$\int x^5 (a + b\text{arccsch}(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.06875, size = 3738, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*((9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e^2)*x^2)*sqrt(e)*log(8*c^4*
e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
```

```
+ (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 + d)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(b*c^3*d*e*x^2 + b*
c^3*d^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x
^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a
*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqr
t(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2 + b*d*e + (9*b*
c^2*d*e + b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqr
t(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e
 + e^2)*x^2 + d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqr
t(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(b*c^3*
d*e*x^2 + b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*
d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*
e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^
2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/24*(32*(b*c^3*d*e*x
^2 + b*c^3*d^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^
2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e
)*x^2 + d^2)) - (9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e^2)*x^2)*sqrt(e)*l
og(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c
^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + e^2) - 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*
x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(2*a*c^3*e^
2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/1
2*(16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 +
2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^
4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e
^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*
sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 +
 d*e)) - 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 + d)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(2*a*c^3*e^2*x^4 - 8
*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{\left(ex^2 + d\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

3.148 $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}}+\frac{2bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e^2\sqrt{-c^2x^2}}$$

```
[Out] (d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cCsch[c*x]))/e^2 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x
^2])])/(e^(3/2)*Sqrt[-(c^2*x^2)]) + (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.275138, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 6302, 12, 573, 157, 63, 217, 203, 93, 204}

$$\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}}+\frac{2bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```


```
[Out] (d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cCsch[c*x]))/e^2 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x
^2])])/(e^(3/2)*Sqrt[-(c^2*x^2)]) + (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e^2*Sqrt[-(c^2*x^2)])
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcx)\int \frac{2d+ex^2}{e^2x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcx)\int \frac{2d+ex^2}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcx)\,\text{Subst}\left(\int \frac{2d+ex}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^{\cdots}\right.}{2e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcdx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, \cdots\right.}{e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{(2bcdx)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^{\cdots}}}\right.}{e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{2bc\sqrt{d}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}} + \frac{(bx)\,\text{S}\cdots}{\cdots}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}} + \frac{2bc\sqrt{d}x\tan\cdots}{e^2\cdots}$$

Mathematica [A] time = 0.369364, size = 233, normalized size = 1.46

$$\frac{\left(2d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} - \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(2c^3\sqrt{d}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right) - \sqrt{c^2}\sqrt{e}\sqrt{c^2d-e}\sqrt{\frac{c^2\left(d+ex^2\right)}{c^2d-e}}\sinh^{-1}\left(\frac{c\cdots}{\cdots}\right.\right.}{c^2e^2\sqrt{c^2x^2+1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*d + e*x^2)*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) - (b*Sqrt[1 + 1/
(c^2*x^2)]*x*(-(Sqrt[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c
^2*d - e)]*ArcSinh[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e]
)]) + 2*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqr
t[-d - e*x^2]]))/(c^2*e^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.467, size = 0, normalized size = 0.

$$\int x^3 (a + b\,\mathrm{arccsch}\,(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.38719, size = 2844, normalized size = 17.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(
c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d
)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)*sq
rt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(b*c*e
*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d
*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d
))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2
```

```
))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 2*(b*c*e*x^2 + 2*b*c*d)*sqr
t(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c*e*x^
2 + b*c*d)*sqrt(d)*log((((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)
*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/
(c*e^3*x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(1/2*((c^3
*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + (b*e*x^2 + b*d)*sqrt(e)*l
og(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c
^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d)
)/(c*e^3*x^2 + c*d*e^2), 1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(1/2*((c
^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2
*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (b*e*x^2 + b*d)*sqrt(-e
)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 2*(b*c*
e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) + 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

3.149 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

```
[Out] -((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(Sqrt[d]*e*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.11148, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {6300, 446, 93, 204}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(Sqrt[d]*e*Sqrt[-(c^2*x^2)])
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{-c^2x^2}}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e\sqrt{-c^2x^2}}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\,\mathrm{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}\\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.15874, size = 122, normalized size = 1.49

$$\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2+1}\sqrt{d+ex^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^2]])/(Sqr
t[d]*e*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.463, size = 0, normalized size = 0.

$$\int x\,(a+b\,\mathrm{arccsch}\,(cx))\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(c^2\int\frac{x}{\left(c^2ex^2+e\right)\sqrt{c^2x^2+1}\sqrt{ex^2+d}+\left(c^2ex^2+e\right)\sqrt{ex^2+d}}\,dx+\frac{\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{ex^2+d}e}+\int\frac{(e\log(c)-e)c^2x^3-\left(c^2d-e\right.}{\left(c^2e^2x^4+\left(c^2de+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -(c^2*integrate(x/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2
*e*x^2 + e)*sqrt(e*x^2 + d)), x) + log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x^2 +
 d)*e) + integrate(((e*log(c) - e)*c^2*x^3 - (c^2*d - e*log(c))*x + (c^2*e*
x^3 + e*x)*log(x))/((c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(e*x^2 +
d)), x))*b - a/(sqrt(e*x^2 + d)*e)
```

Fricas [B] time = 3.04869, size = 822, normalized size = 10.02

$$\left[-\frac{4\sqrt{ex^2+d}bd\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+4\sqrt{ex^2+d}ad-\left(bex^2+bd\right)\sqrt{d}\log\left(\frac{\left(c^4d^2+6c^2de+e^2\right)x^4+8\left(c^2d^2+de\right)x^2+4\left(\left(c^3d+ce\right)x^3+2cdx\right)\sqrt{ex^2+\right.}{x^4}\right.}{4\left(de^2x^2+d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 + 6*c^
2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*
sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4))/(d*e^2
*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d + (b*e*x^2 + b*d)*sqrt(-d)*arcta
n(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2
+ 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)))/(d*e^2*x^2 + d^
2*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

3.150 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi [A] time = 0.120881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 31.6475, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [A] time = 0.453, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arccsch}(cx)}{x} \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{e^2x^5 + 2\,dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

3.151 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi [A] time = 0.132713, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]

[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 37.1715, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [A] time = 0.451, size = 0, normalized size = 0.

$$\int \frac{a + b\text{arccsch}\,(cx)}{x^3} \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\text{arcsch}\,(cx) + a)}{e^2x^7 + 2\,dex^5 + d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

3.152 $\int \frac{x^4\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.111334, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]

[Out] Defer[Int][(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 8.15887, size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.477, size = 0, normalized size = 0.

$$\int x^4 (a + b\text{arccsch}(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^4 \operatorname{arcsch}(cx) + ax^4\right)\sqrt{ex^2 + d}}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcsch}(cx)+a)x^4}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

3.153 $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.107016, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.44414, size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.448, size = 0, normalized size = 0.

$$\int x^2 (a + b\operatorname{arccsch}(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 \operatorname{arcsch}(cx) + ax^2\right)\sqrt{ex^2 + d}}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

3.154 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}$$

```
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[d + e*x^2]*Ellipti
cF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sq
rt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rubi [A] time = 0.075112, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {191, 6292, 12, 418}

$$\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}\,F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[d + e*x^2]*Ellipti
cF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sq
rt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6292

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
```

```
1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bcx)\int \frac{1}{d\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}}\\ &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bcx)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}}\\ &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d\left(1+c^2x^2\right)}}}\end{aligned}$$

Mathematica [A] time = 0.192663, size = 113, normalized size = 1.02

$$\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-c^2}x\right),\frac{e}{c^2d}\right)}{\sqrt{-c^2}d\sqrt{c^2x^2+1}\sqrt{d+ex^2}}+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x
*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[Sqrt[-c^2]*x], e/(c^2*d)])/(Sqrt[-c^2
]*d*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{arccsch}\,(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx + \frac{ax}{\sqrt{ex^2 + d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x) + a*
x/(sqrt(e*x^2 + d)*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.155 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=321

$$\frac{2bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bc\sqrt{-}}{}$$

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*
c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]) - (a + b*ArcCs
ch[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsch[c*x]))/(d^2*Sqrt[d +
 e*x^2]) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/
(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))
]) + (2*b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^3*S
qrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rubi [A] time = 0.319709, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {271, 191, 6302, 12, 583, 531, 418, 492, 411}

$$-\frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}+\frac{2bex\sqrt{d+ex^2}F\left(\tan^{-1}(c\right.}{d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```


```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*
c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]) - (a + b*ArcCs
ch[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsch[c*x]))/(d^2*Sqrt[d +
 e*x^2]) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/
(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))
]) + (2*b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^3*S
qrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```


Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-d-2ex^2}{d^2x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-d-2ex^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2de-c^2dex^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{(2bcex)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}}{d^2\sqrt{-c^2x^2}}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} +$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} -$$

Mathematica [C] time = 0.435116, size = 201, normalized size = 0.63

$$\frac{-a\left(d+2ex^2\right)+bcx\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)-b\operatorname{csch}^{-1}(cx)\left(d+2ex^2\right)}{d^2x\sqrt{d+ex^2}}+\frac{ibcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}\left(\left(2e-c^2d\right)\text{EllipticF}\left(i\sinh\right.\right.}{\sqrt{c^2}d^2\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^2
)*ArcCsch[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
rt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-(
c^2*d) + 2*e)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.455, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}\left(cx\right)}{x^2}\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{e^2x^6+2\,dex^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcsch}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

3.156 $\int \frac{x^5\left(a+b\textbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{bcdx\sqrt{-c^2x^2-1}}{3e^2\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{8bc\sqrt{}}{}$$

```
[Out] (b*c*d*x*Sqrt[-1 - c^2*x^2])/(3*(c^2*d - e)*e^2*Sqrt[-(c^2*x^2)]*Sqrt[d + e
*x^2]) - (d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b
*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x
]))/e^3 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(e
^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 1.22935, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 6302, 12, 1614, 157, 63, 217, 203, 93, 204}

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{bcdx\sqrt{-c^2x^2-1}}{3e^2\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{8bc\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*x*Sqrt[-1 - c^2*x^2])/(3*(c^2*d - e)*e^2*Sqrt[-(c^2*x^2)]*Sqrt[d + e
*x^2]) - (d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b
*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x
]))/e^3 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(e
^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\int \frac{}{3e^3}}{}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\int \frac{8}{x}}{3}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\,\text{Sub}}{}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{}$$

Mathematica [A] time = 0.556475, size = 327, normalized size = 1.3

$$\frac{a\left(c^2d-e\right)\left(8d^2+12dex^2+3e^2x^4\right)+b\left(c^2d-e\right)\text{csch}^{-1}(cx)\left(8d^2+12dex^2+3e^2x^4\right)+bcdex\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)}{3e^3\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{bx\sqrt{\frac{1}{c^2x}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d - e)*(8*d^2 + 12*d*
e*x^2 + 3*e^2*x^4) + b*(c^2*d - e)*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcCsch
[c*x])/(3*(c^2*d - e)*e^3*(d + e*x^2)^(3/2)) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(
-3*Sqrt[c^2]*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d - e)]*Ar
cSinh[(c*Sqrt[e]*Sqrt[1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d - e])] + 8*c^3*Sq
rt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^2]]
))/(3*c^2*e^3*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.466, size = 0, normalized size = 0.

$$\int x^5 (a + b\operatorname{arccsch}(cx)) \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.79753, size = 5065, normalized size = 20.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e
- b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*
e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt
(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3
*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2
 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(b*c^3*d^3 - b
*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*s
qrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c
^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e
^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e
*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 - c*d^2*e^
4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), -1/6*(3*(b*c^
2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2
)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e
)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e))
 - 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 1)/(c*x)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 +
2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x
^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d
)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(8*a*c^3*d^3 - 8*
a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^
2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e
*x^2 + d))/(c^3*d^3*e^3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*
e^4 - c*d*e^5)*x^2), 1/12*(16*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e
^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*
e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c
^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*
e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x
^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^
4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2
) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^
3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 1)/(c*x)) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3
)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 - c*d^2*e^4
+ (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), 1/6*(8*(b*c^3*d
^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*
x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt
(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2
)) - 3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e -
b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2
+ d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*
x^2 + d*e)) + 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4
```

```
+ 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2
 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c
^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 -
c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

3.157 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{2bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}}-\frac{bcx\sqrt{-c^2x^2-1}}{3e\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[Out] -(b*c*x*Sqrt[-1 - c^2*x^2])/(3*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x^
2]) + (d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsch[c
*x])/(e^2*Sqrt[d + e*x^2]) - (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[
-1 - c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.268162, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {266, 43, 6302, 12, 573, 152, 93, 204}

$$-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{2bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}}-\frac{bcx\sqrt{-c^2x^2-1}}{3e\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```


```
[Out] -(b*c*x*Sqrt[-1 - c^2*x^2])/(3*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x^
2]) + (d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsch[c
*x])/(e^2*Sqrt[d + e*x^2]) - (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[
-1 - c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[-(c^2*x^2)])
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{3e^2x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\,\text{Subst}\left(\int \frac{-2d-3ex}{x\sqrt{-1-c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e^2\sqrt{-c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\,\text{Subst}\left(\int \right.}{3d\left(c^2\right.}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\,\text{Subst}\left(\int \right.}{3}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(2bcx)\,\text{Subst}\left(\right.}{3}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\,\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\tan^{-1}\left(\frac{}{\sqrt{d}}\right.}{3\sqrt{d}e^2\sqrt{-}}$$

Mathematica [A] time = 0.284124, size = 201, normalized size = 1.19

$$\frac{a\left(c^2d-e\right)\left(2d+3ex^2\right)+bcex\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)+b\left(c^2d-e\right)\text{csch}^{-1}(cx)\left(2d+3ex^2\right)}{3e^2\left(e-c^2d\right)\left(d+ex^2\right)^{3/2}} + \frac{2bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-d-ex^2}\tan^{-1}}{3\sqrt{d}e^2\sqrt{c^2x^2+1}\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d - e)*(2*d + 3*e*x^2)
+ b*(c^2*d - e)*(2*d + 3*e*x^2)*ArcCsch[c*x])/(3*e^2*(-(c^2*d) + e)*(d + e*
x^2)^(3/2)) + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[
d]*Sqrt[1 + c^2*x^2])/Sqrt[-d - e*x^2]])/(3*Sqrt[d]*e^2*Sqrt[1 + c^2*x^2]*S
qrt[d + e*x^2])
```

Maple [F] time = 0.5, size = 0, normalized size = 0.

$$\int x^{3} \left(a + b\,\mathrm{arccsch}\left(c x\right)\right) \left(e x^{2}+d\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\,a\left(\frac{3\,x^{2}}{\left(e x^{2}+d\right)^{\frac{3}{2}} e}+\frac{2\,d}{\left(e x^{2}+d\right)^{\frac{3}{2}} e^{2}}\right)+b\int \frac{x^{3} \log\left(\sqrt{\frac{1}{c^{2} x^{2}}+1}+\frac{1}{c x}\right)}{\left(e x^{2}+d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*inte
grate(x^3*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Fricas [B] time = 4.10332, size = 1627, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (
b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(d)
*log((((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d +
 c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 8*d^2)/x^4) + 2*(2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2
+ (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 +
 d))/(c^2*d^4*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3 -
d^2*e^4)*x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(
b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*
x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c
^2*d^2 + d*e)*x^2 + d^2)) + (2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d
*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + (2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c*d*e^2*
x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d^4
*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3 - d^2*e^4)*x^2)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

3.158 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}+\frac{bcx\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x^2
]) - (a + b*ArcCsch[c*x])/(3*e*(d + e*x^2)^(3/2)) - (b*c*x*ArcTan[Sqrt[d +
e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*d^(3/2)*e*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.143888, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6300, 446, 96, 93, 204}

$$-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}+\frac{bcx\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```


```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x^2
]) - (a + b*ArcCsch[c*x])/(3*e*(d + e*x^2)^(3/2)) - (b*c*x*ArcTan[Sqrt[d +
e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*d^(3/2)*e*Sqrt[-(c^2*x^2)])
```


Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
 x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6de\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3de\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}$$

Mathematica [A] time = 0.235849, size = 185, normalized size = 1.28

$$\frac{ad\left(e-c^2d\right)+bcex\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)+bd\left(e-c^2d\right)\text{csch}^{-1}(cx)}{3de\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}+\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2+1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (a*d*(-(c^2*d) + e) + b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) + b*d*(-(c^
2*d) + e)*ArcCsch[c*x])/(3*d*(c^2*d - e)*e*(d + e*x^2)^(3/2)) + (b*c*Sqrt[1
 + 1/(c^2*x^2)]*x*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[
-d - e*x^2]])/(3*d^(3/2)*e*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.496, size = 0, normalized size = 0.

$$\int x\left(a+b\text{arccsch}\left(cx\right)\right)\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{x\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx-\frac{a}{3\left(ex^2+d\right)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x) -
1/3*a/((e*x^2 + d)^(3/2)*e)
```

Fricas [B] time = 3.91455, size = 1439, normalized size = 9.99

$$\left[-\frac{4\left(bc^2d^3-bd^2e\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(bc^2d^3+\left(bc^2de^2-be^3\right)x^4-bd^2e+2\left(bc^2d^2e-bde^2\right)x^2\right)\sqrt{d}\log\left(\frac{\left(c^4d^2+6\right.}{}\right.}{12\left(c^2d^5e-d^4e^2+\left(c^2d^3e^3-d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(b*c^2*d^3 - b*d^2*e)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e
+ 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x
^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d
)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c^2*d^3 - a*d^
2*e - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x
```

```
^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^
2 - d^3*e^3)*x^2), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e +
 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
+ (c^2*d^2 + d*e)*x^2 + d^2)) + 2*(b*c^2*d^3 - b*d^2*e)*sqrt(e*x^2 + d)*log
((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(a*c^2*d^3 - a*d^2*e -
(b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d
))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^
3*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

3.159 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]

Rubi [A] time = 0.124076, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]

[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 42.6218, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [A] time = 0.456, size = 0, normalized size = 0.

$$\int \frac{a + b\mathrm{arccsch}\,(cx)}{x} \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{e^3x^7 + 3\,de^2x^5 + 3\,d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

3.160 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\,\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi [A] time = 0.152306, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]

[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 55.2559, size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [A] time = 0.483, size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^3}\left(ex^2 + d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)}{e^3x^9 + 3\,de^2x^7 + 3\,d^2ex^5 + d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

3.161 $\int \frac{x^6\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^6\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi [A] time = 0.131673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^6\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]

[Out] Defer[Int][(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^6\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 12.5662, size = 0, normalized size = 0.

$$\int \frac{x^6\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A] time = 0.491, size = 0, normalized size = 0.

$$\int x^{6} \left(a + b \operatorname{arccsch}\left(cx\right)\right) \left(e x^{2} + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b x^{6} \operatorname{arcsch}\left(cx\right) + a x^{6}\right) \sqrt{e x^{2} + d}}{e^{3} x^{6} + 3\, d e^{2} x^{4} + 3\, d^{2} e x^{2} + d^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arccsch(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^6}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

3.162 $\int \frac{x^4\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi [A] time = 0.109875, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]

[Out] Defer[Int][(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 12.2303, size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A] time = 0.489, size = 0, normalized size = 0.

$$\int x^{4}\left(a+b\operatorname{arccsch}\left(cx\right)\right)\left(ex^{2}+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^{4}\operatorname{arcsch}\left(cx\right)+ax^{4}\right)\sqrt{ex^{2}+d}}{e^{3}x^{6}+3\,de^{2}x^{4}+3\,d^{2}ex^{2}+d^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

3.163 $\int \frac{x^2\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{3d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{x^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3de\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)}+\frac{bcx^2\sqrt{}}{3d\sqrt{-c^2x^2}\left(c^2\right.}$$

```
[Out] (b*c*x^2*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x
^2]) + (b*c^3*x^2*Sqrt[d + e*x^2])/(3*d*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2]) + (x^3*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c
^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d*(c^2*d - e
)*e*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]
) + (b*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^2*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rubi [A] time = 0.307725, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {264, 6302, 12, 471, 422, 418, 492, 411}

$$\frac{x^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bx\sqrt{d+ex^2}\,F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{3d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3de\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)}+\frac{bcx^2\sqrt{}}{3d\sqrt{-c^2x^2}\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```


```
[Out] (b*c*x^2*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x
^2]) + (b*c^3*x^2*Sqrt[d + e*x^2])/(3*d*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2]) + (x^3*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c
^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d*(c^2*d - e
)*e*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]
) + (b*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^2*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{(bcx)\int\frac{x^2}{3d\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$=\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{(bcx)\int\frac{x^2}{\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d\sqrt{-c^2x^2}}$$

$$=\frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}+\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\int\frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d\left(-c^2d+e\right)\sqrt{-c^2x^2}}$$

$$=\frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}+\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{(bcx)\int\frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d\left(-c^2d+e\right)\sqrt{-c^2x^2}}-\frac{(bc^3 \cdots}{3d \cdots}$$

$$=\frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} \cdots$$

$$=\frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} \cdots$$

Mathematica [A] time = 0.280522, size = 189, normalized size = 0.53

$$\frac{x^2\left(ax\left(c^2d-e\right)+bc\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)+bx\left(c^2d-e\right)\operatorname{csch}^{-1}(cx)\right)}{3d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{e}{d}}x\right)|\frac{c^2d}{e}\right)}{3d\sqrt{c^2x^2+1}\sqrt{-\frac{e}{d}}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d - e)*x + b*c*Sqrt[1 + 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d -
e)*x*ArcCsch[c*x]))/(3*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 + 1/(
c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], (c^2*d)/e
])/(3*d*(c^2*d - e)*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int x^2\left(a+b\operatorname{arccsch}(cx)\right)\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}a\left(\frac{x}{\left(ex^2+d\right)^{\frac{3}{2}}e}-\frac{x}{\sqrt{ex^2+d}de}\right)+b\int\frac{x^2\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 \operatorname{arcsch}(cx) + ax^2\right)\sqrt{ex^2+d}}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

3.164 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=278

$$-\frac{bx\left(3c^2d-2e\right)\sqrt{d+ex^2}\text{EllipticF}\left(\tan^{-1}(cx),1-\frac{e}{c^2d}\right)}{3d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\sqrt{e}x\sqrt{-c^2x^2 ...}}{3d^{3/2}\sqrt{-c^2x^2}\left(c ...\right.}$$

```
[Out] (x*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsch[c*x]
))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*Sqrt[e]*x*Sqrt[-1 - c^2*x^2]*EllipticE[Ar
cTan[(Sqrt[e]*x)/Sqrt[d]], 1 - (c^2*d)/e])/(3*d^(3/2)*(c^2*d - e)*Sqrt[-(c^
2*x^2)]*Sqrt[(d*(1 + c^2*x^2))/(d + e*x^2)]*Sqrt[d + e*x^2]) - (b*(3*c^2*d
- 2*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^3*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rubi [A] time = 0.176579, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {192, 191, 6292, 12, 525, 418, 411}

$$\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bx\left(3c^2d-2e\right)\sqrt{d+ex^2}F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{3d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{bc\sqrt{e}x\sqrt{-c^2x^2-1}E\left(\tan^{-1} ...\right.}{3d^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d ...}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```


```
[Out] (x*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsch[c*x]
))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*Sqrt[e]*x*Sqrt[-1 - c^2*x^2]*EllipticE[Ar
cTan[(Sqrt[e]*x)/Sqrt[d]], 1 - (c^2*d)/e])/(3*d^(3/2)*(c^2*d - e)*Sqrt[-(c^
2*x^2)]*Sqrt[(d*(1 + c^2*x^2))/(d + e*x^2)]*Sqrt[d + e*x^2]) - (b*(3*c^2*d
- 2*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^3*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6292

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{3d+2ex^2}{3d^2\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$=\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{3d+2ex^2}{\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d^2\sqrt{-c^2x^2}}$$

$$=\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{\left(bc\left(3c^2d-2e\right)x\right)\int\frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}}-\frac{(bcex)\int}{3d\left(c^2d\right.}$$

$$=\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{bc\sqrt{ex}\sqrt{-1-c^2x^2}E\left(\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{\frac{d\left(1+c^2x^2\right)}{d+ex^2}}\sqrt{d+ex^2}}-\frac{b\left(\right.}{3d}$$

Mathematica [C] time = 0.532478, size = 248, normalized size = 0.89

$$\frac{x\left(a\left(c^2d-e\right)\left(3d+2ex^2\right)-bcex\sqrt{\frac{1}{c^2x^2}+1}\left(d+ex^2\right)+b\left(c^2d-e\right)\operatorname{csch}^{-1}(cx)\left(3d+2ex^2\right)\right)}{3d^2\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{ibcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}+1}\left(2\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d - e)*(3*d + 2*e
*x^2) + b*(c^2*d - e)*(3*d + 2*e*x^2)*ArcCsch[c*x]))/(3*d^2*(c^2*d - e)*(d
+ e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c
^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + 2*(c^2*d - e)*EllipticF
[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*(c^2*d - e)*Sqrt[1 + c
^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.483, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccsch}(cx))\left(ex^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a\left(\frac{2x}{\sqrt{ex^2+d}d^2}+\frac{x}{\left(ex^2+d\right)^{\frac{3}{2}}d}\right)+b\int\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^
2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcsch}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.165 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=596

$$-\frac{bx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{c^6d^3(m+2)(m+4)(m+6)}{m+1}-\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)-3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{(m+3)(m+5)(m+7)}\right)\text{H}}{c^5f(m+1)(m+2)(m+4)(m+6)\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 - c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^
2)]) - (b*e^2*(e*(5 + m)^2 - 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqr
t[-1 - c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^2)])
 + (b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 - c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[-
(c^2*x^2)]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (3*d^2
*e*(f*x)^(3 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 +
m)*(a + b*ArcCsch[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsch[
c*x]))/(f^7*(7 + m)) - (b*((c^6*d^3*(2 + m)*(4 + m)*(6 + m))/(1 + m) - (e*(
1 + m)*(e^2*(15 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*
c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/((3 + m)*(5 + m)*(7 + m)))
*x*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m
)/2, -(c^2*x^2)])/(c^5*f*(1 + m)*(2 + m)*(4 + m)*(6 + m)*Sqrt[-(c^2*x^2)]*S
qrt[-1 - c^2*x^2])
```

Rubi [A] time = 2.57497, antiderivative size = 577, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {270, 6302, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{cs}\right.}{f^7(m+7}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```


```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 - c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^
2)]) - (b*e^2*(e*(5 + m)^2 - 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqr
```

```
t[-1 - c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^2)])
 + (b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 - c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[-
(c^2*x^2)]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (3*d^2
*e*(f*x)^(3 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 +
m)*(a + b*ArcCsch[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsch[
c*x]))/(f^7*(7 + m)) - (b*c*(d^3/(1 + m)^2 - (e*(e^2*(15 + 8*m + m^2)^2 - 3
*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 2
2*m^3 + m^4)))/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*
x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -
(c^2*x^2)])/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
```

```
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{d^3(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{3de^2(fx)^{5+}}{}$$

$$= \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a\right.}{f^3(3}$$

$$= -\frac{be^2\left(e(5+m)^2-3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{be^3x(fx)^5}{cf^5(6+m)}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638\right.\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638\right.\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638\right.\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(}$$

Mathematica [F] time = 0.212125, size = 0, normalized size = 0.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]), x]
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right)^3 \left(a+b\,\text{arccsch}\,(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\text{arcsch}\,(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^3 (b \operatorname{arcsch}(cx)+a) \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\textbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=379

$$-\frac{bx\sqrt{c^2x^2+1}(fx)^{m+1}\left(c^4d^2(m+2)(m+3)(m+4)(m+5)+e(m+1)^2\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)\right)\text{Hypergeo}}{c^3f(m+1)^2(m+2)(m+3)(m+4)(m+5)\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

```
[Out] -((b*e*(e*(3 + m)^2 - 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 - c
^2*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)])) + (b*e^2
*x*(f*x)^(3 + m)*Sqrt[-1 - c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)
]) + (d^2*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3
 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCs
ch[c*x]))/(f^5*(5 + m)) - (b*(c^4*d^2*(2 + m)*(3 + m)*(4 + m)*(5 + m) + e*(
1 + m)^2*(e*(3 + m)^2 - 2*c^2*d*(20 + 9*m + m^2)))*x*(f*x)^(1 + m)*Sqrt[1 +
 c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(c^3*f*
(1 + m)^2*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^
2])
```

Rubi [A] time = 0.491282, antiderivative size = 360, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {270, 6302, 12, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}-\frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 - 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 - c
^2*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)])) + (b*e^2
*x*(f*x)^(3 + m)*Sqrt[-1 - c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)
]) + (d^2*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3
 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCs
ch[c*x]))/(f^5*(5 + m)) - (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 - 2*c^2*d*(
20 + 9*m + m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sq
rt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(
f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\cdots}{f\cdots}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\cdots}{f\cdots}$$

$$= \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(\cdots\right.}{f^3(\cdots}$$

$$= -\frac{be\left(e(3+m)^2-2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}} + \frac{be^2x(fx)^{3\cdots}}{cf^3(4+m)\cdots}$$

$$= -\frac{be\left(e(3+m)^2-2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}} + \frac{be^2x(fx)^{3\cdots}}{cf^3(4+m)\cdots}$$

$$= -\frac{be\left(e(3+m)^2-2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}} + \frac{be^2x(fx)^{3\cdots}}{cf^3(4+m)\cdots}$$

Mathematica [F] time = 0.139643, size = 0, normalized size = 0.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]), x]
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(e x^{2}+d\right)^{2} \left(a+b \,\mathrm{arccsch}\left(c x\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a e^{2} x^{4}+2 \, a d e x^{2}+a d^{2}+\left(b e^{2} x^{4}+2 \, b d e x^{2}+b d^{2}\right) \mathrm{arcsch}\left(c x\right)\right) \left(f x\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 (b\operatorname{arcsch}(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.167 $\int (fx)^m \left(d+ex^2\right)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=220

$$\frac{bx\sqrt{c^2x^2+1}(fx)^{m+1}\left(e(m+1)^2-c^2d(m+2)(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)}{cf(m+1)^2(m+2)(m+3)\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{d(fx)^{m+1}\left(a+b\,\mathrm{csch}\right.}{f(m+1)}$$

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[-(c^2*x^
2)]) + (d*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m
)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (b*(e*(1 + m)^2 - c^2*d*(2 + m)*(3
+ m))*x*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)])/(c*f*(1 + m)^2*(2 + m)*(3 + m)*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2])
```

Rubi [A] time = 0.21957, antiderivative size = 208, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 6302, 12, 459, 365, 364}

$$\frac{d(fx)^{m+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{f^3(m+3)}-\frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{d}{(m+1)^2}-\frac{e}{c^2(m+2)(m+3)}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\right.}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[-(c^2*x^
2)]) + (d*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m
)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) - (b*c*(d/(1 + m)^2 - e/(c^2*(2 + m)*
(3 + m)))*x*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)dx = \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} - \frac{(bcx)\int \frac{(fx)^m(d(3+}{(1+m)(3+}}{\sqrt{-c}}$$

$$= \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} - \frac{(bcx)\int \frac{(fx)^m(d(3+}{\sqrt{-}}}{(3+4m+m}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+bcs\right.}{f^3(3+m}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+bcs\right.}{f^3(3+m}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+bcs\right.}{f^3(3+m}$$

Mathematica [F] time = 0.104974, size = 0, normalized size = 0.

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]), x]
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right)\left(a+b\,\text{arccsch}\,(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^2 + ad + \left(bex^2 + bd\right)\operatorname{arcsch}(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)\left(b\operatorname{arcsch}(cx) + a\right)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.168 $\int \frac{(fx)^m\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Rubi [A] time = 0.0785348, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A] time = 1.79067, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Maple [A] time = 0.519, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccsch}\left(cx\right)\right)}{e\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arcsch}\left(cx\right)+a\right)\left(fx\right)^{m}}{e\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\,\mathrm{arcsch}\left(cx\right)+a\right)\left(fx\right)^{m}}{e\,x^{2}+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)(fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.169 $\int \frac{(fx)^m\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.0760004, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 2.71711, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Maple [A] time = 0.501, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b\,\mathrm{arccsch}\,(cx))}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)\,(fx)^m}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arcsch}\,(cx) + a)\,(fx)^m}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a)\,(fx)^m}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.170 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\,\mathbf{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.116284, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Mathematica [A] time = 1.08432, size = 0, normalized size = 0.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.484, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(e x^{2}+d\right)^{\frac{3}{2}} \left(a+b \operatorname{arccsch}\left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^{2}+d\right)^{\frac{3}{2}} \left(b \operatorname{arcsch}\left(cx\right)+a\right) \left(fx\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(aex^{2}+ad+\left(bex^{2}+bd\right) \operatorname{arcsch}\left(cx\right)\right) \sqrt{ex^{2}+d} \left(fx\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.171 $\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt{d+ex^2}(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi [A] time = 0.104174, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx=\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 0.106652, size = 0, normalized size = 0.

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [A] time = 0.483, size = 0, normalized size = 0.

$$\int (fx)^m (a + b\,\mathrm{arccsch}\,(cx)) \sqrt{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)\,(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a)\,(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b\operatorname{arcsch}(cx) + a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.172 $\int \frac{(fx)^m\left(a+b\,\textbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.103437, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.44761, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.486, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(a+b\,\mathrm{arccsch}\left(cx\right)\right) \frac{1}{\sqrt{e x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arcsch}\left(cx\right)+a\right)\left(fx\right)^{m}}{\sqrt{e x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\,\mathrm{arcsch}\left(cx\right)+a\right)\left(fx\right)^{m}}{\sqrt{e x^{2}+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)\left(fx\right)^m}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.173 $\int \frac{(fx)^m\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.114956, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 1.71656, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.48, size = 0, normalized size = 0.

$$\int (fx)^m (a + b\,\mathrm{arccsch}\,(cx)) \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx) + a) (fx)^m}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2 + d}(b\,\mathrm{arcsch}\,(cx) + a) (fx)^m}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.174 $\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{c^2x^2+1}\,(1-c}{90c^{13}x\sqrt{\frac{1}{c^2x^2}}}$$

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x
) + (7*b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqrt[1 + 1/(c^2*x^
2)]*x) - (13*b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(150*c^13*Sqrt[1 + 1/
(c^2*x^2)]*x) + (3*b*(1 - c^2*x^2)^(7/2)*Sqrt[1 + c^2*x^2])/(70*c^13*Sqrt[1
 + 1/(c^2*x^2)]*x) - (b*(1 - c^2*x^2)^(9/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqr
t[1 + 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^12) +
 ((1 - c^4*x^4)^(3/2)*(a + b*ArcCsch[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)
*(a + b*ArcCsch[c*x]))/(10*c^12) + (4*b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 -
c^2*x^2]])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rubi [A] time = 2.3444, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {266, 43, 6310, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{c^2x^2+1}\,(1-c}{90c^{13}x\sqrt{\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x
) + (7*b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqrt[1 + 1/(c^2*x^
2)]*x) - (13*b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(150*c^13*Sqrt[1 + 1/
(c^2*x^2)]*x) + (3*b*(1 - c^2*x^2)^(7/2)*Sqrt[1 + c^2*x^2])/(70*c^13*Sqrt[1
 + 1/(c^2*x^2)]*x) - (b*(1 - c^2*x^2)^(9/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqr
t[1 + 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^12) +
 ((1 - c^4*x^4)^(3/2)*(a + b*ArcCsch[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)
*(a + b*ArcCsch[c*x]))/(10*c^12) + (4*b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 -
c^2*x^2]])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csc}\right.}{10c^{12}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{13b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{3b\left(\right.}{}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{13b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{3b\left(\right.}{}$$

Mathematica [A] time = 0.309092, size = 214, normalized size = 0.54

$$-\frac{105a\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)+\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{1-c^4x^4}\left(35c^8x^8-5c^6x^6+78c^4x^4-36c^2x^2+768\right)}{c^2x^2+1}+840b\log\left(c^2x^3+x\right)-840b\log(\ldots}{3150c^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1 + 1/(c^
2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 - 36*c^2*x^2 + 78*c^4*x^4 - 5*c^6*x^6 + 35
*c^8*x^8))/(1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x
^8)*ArcCsch[c*x] + 840*b*Log[x + c^2*x^3] - 840*b*Log[1 + c^2*x^2 + c*Sqrt[
1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/(3150*c^12)
```

Maple [F] time = 0.483, size = 0, normalized size = 0.

$$\int x^{11}\left(a+b\operatorname{arccsch}(cx)\right)\frac{1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{30}a\left(\frac{3\left(-c^4x^4+1\right)^{\frac{5}{2}}}{c^{12}}-\frac{10\left(-c^4x^4+1\right)^{\frac{3}{2}}}{c^{12}}+\frac{15\sqrt{-c^4x^4+1}}{c^{12}}\right)+\frac{1}{30}b\left(\frac{\left(3c^{12}x^{12}+c^8x^8+4c^4x^4-8\right)\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}c^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*lo
```

```
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^
12) - 30*integrate((x^11*log(c) + x^11*log(x))*e^(-1/2*log(c^2*x^2 + 1) - 1
/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 30*integrate(1/30*(3*c^10*x^11 -
 3*c^8*x^9 + 4*c^6*x^7 - 4*c^4*x^5 + 8*c^2*x^3 - 8*x)/(sqrt(c^2*x^2 + 1)*sq
rt(c*x + 1)*sqrt(-c*x + 1)*c^10 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^10), x))
```

Fricas [A] time = 2.68003, size = 846, normalized size = 2.14

$$-\frac{105\left(3\,bc^{10}x^{10}+3\,bc^{8}x^{8}+4\,bc^{6}x^{6}+4\,bc^{4}x^{4}+8\,bc^{2}x^{2}+8\,b\right)\sqrt{-c^{4}x^{4}+1}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+\left(35\,bc^{9}x^{9}-5\,bc^{7}x^{7}+78\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3150*(105*(3*b*c^10*x^10 + 3*b*c^8*x^8 + 4*b*c^6*x^6 + 4*b*c^4*x^4 + 8*b
*c^2*x^2 + 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + (35*b*c^9*x^9 - 5*b*c^7*x^7 + 78*b*c^5*x^5 - 36*b*c^3*x^3 + 76
8*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 420*(b*c^2*x^2
+ b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c^2*x^2 + 1)) + 420*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 105*(3*a*c^10*x^10
+ 3*a*c^8*x^8 + 4*a*c^6*x^6 + 4*a*c^4*x^4 + 8*a*c^2*x^2 + 8*a)*sqrt(-c^4*x^
4 + 1))/(c^14*x^2 + c^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^{11}}{\sqrt{-c^4x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^11/sqrt(-c^4*x^4 + 1), x)
```

3.175 $\int \frac{x^7\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=264

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{5/2}}{30c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{3/2}}{18c^9x\sqrt{\frac{1}{c^2x^2}+1}}-$$

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[1 + 1/(c^2*x^2)]*x) +
(b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(18*c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
- (b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(30*c^9*Sqrt[1 + 1/(c^2*x^2)]*x
) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)
*(a + b*ArcCsch[c*x]))/(6*c^8) + (b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 - c^2*
x^2]])/(3*c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rubi [A] time = 1.91615, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {266, 43, 6310, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{5/2}}{30c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{3/2}}{18c^9x\sqrt{\frac{1}{c^2x^2}+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```


```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[1 + 1/(c^2*x^2)]*x) +
(b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(18*c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
- (b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(30*c^9*Sqrt[1 + 1/(c^2*x^2)]*x
) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)
*(a + b*ArcCsch[c*x]))/(6*c^8) + (b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 - c^2*
x^2]])/(3*c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{6c^8\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{6c^9}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{\left(-2-c^4\right.}{x\sqrt{}}}{6c^9\sqrt{1+\frac{1}{c^2x}}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\right.}{12c^9\sqrt{}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\right.}{1}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\right.}{6c^9\sqrt{1+}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\right.}{6c^9\sqrt{1+}}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4}}{}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4}}{}$$

Mathematica [A] time = 0.407412, size = 180, normalized size = 0.68

$$-\frac{15a\sqrt{1-c^4x^4}\left(c^4x^4+2\right)+\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{1-c^4x^4}\left(3c^4x^4-c^2x^2+28\right)}{c^2x^2+1}+30b\log\left(c^2x^3+x\right)-30b\log\left(c^2x^2+cx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{1-c^4x^4}\right.}{90c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
1 - c^4*x^4]*(28 - c^2*x^2 + 3*c^4*x^4))/(1 + c^2*x^2) + 15*b*Sqrt[1 - c^4*
x^4]*(2 + c^4*x^4)*ArcCsch[c*x] + 30*b*Log[x + c^2*x^3] - 30*b*Log[1 + c^2*
x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/(90*c^8)
```

Maple [F] time = 0.423, size = 0, normalized size = 0.

$$\int x^7\left(a+b\operatorname{arccsch}(cx)\right)\frac{1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}a\left(\frac{\left(-c^4x^4+1\right)^{\frac{3}{2}}}{c^8}-\frac{3\sqrt{-c^4x^4+1}}{c^8}\right)+\frac{1}{6}b\left(\frac{\left(c^8x^8+c^4x^4-2\right)\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}c^8}-6\int\left(x^7\log(c)+x^7\log(x)\right)e^{\left(-\frac{1}{2}\log(c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*x
^8 + c^4*x^4 - 2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x +
```

```
1)*sqrt(-c*x + 1)*c^8) - 6*integrate((x^7*log(c) + x^7*log(x))*e^(-1/2*log(
c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 6*integrate(1/6*
(c^6*x^7 - c^4*x^5 + 2*c^2*x^3 - 2*x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*c^6 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^6), x))
```

Fricas [A] time = 2.77709, size = 699, normalized size = 2.65

$$-\frac{15\left(bc^6x^6+bc^4x^4+2\,bc^2x^2+2\,b\right)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(3\,bc^5x^5-bc^3x^3+28\,bcx\right)\sqrt{-c^4x^4+1}\sqrt{\frac{c^2x^2+1}{c^2x^2}}-15\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(15*(b*c^6*x^6 + b*c^4*x^4 + 2*b*c^2*x^2 + 2*b)*sqrt(-c^4*x^4 + 1)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (3*b*c^5*x^5 - b*c^3*x^3
 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 15*(b*c^2*x
^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 1)/(c^2*x^2 + 1)) + 15*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1
)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 15*(a*c^6*x^6 + a
*c^4*x^4 + 2*a*c^2*x^2 + 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 + c^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^7/sqrt(-c^4*x^4 + 1), x)
```

3.176 $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{2c^4}+\frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}-\frac{bx\tan^{-1}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-c^2x^2-1}}\right)}{2c^3\sqrt{-c^2x^2}}$$

```
[Out] (b*x*Sqrt[1 - c^4*x^4])/(2*c^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) - (Sqrt
[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^4) - (b*x*ArcTan[Sqrt[1 - c^4*x^4]
/Sqrt[-1 - c^2*x^2]])/(2*c^3*Sqrt[-(c^2*x^2)])
```

Rubi [A] time = 0.214117, antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {261, 6310, 12, 1572, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{2c^4}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{b\sqrt{c^2x^2+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(2*c^5*Sqrt[1 + 1/(c^2*x^2)]*x) -
(Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^4) + (b*Sqrt[1 + c^2*x^2]*Arc
Tanh[Sqrt[1 - c^2*x^2]])/(2*c^5*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
```

```
FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1572

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*Frac
Part[q])*(1 + d/(x^mn*e))^FracPart[q]), Int[x^(m + mn*q)*(1 + d/(x^mn*e))^q
*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && EqQ[n2, -
2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}+\frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{2c^5}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1+c^2x^2}}\,dx}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^2x}}{x}\,dx,x,x^2\right)}{4c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\right.}{4c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}+\frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,d\right.}{2c^7\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}+\frac{b\sqrt{1+c^2x^2}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

Mathematica [A] time = 0.33708, size = 141, normalized size = 1.08

$$-\frac{a\sqrt{1-c^4x^4}+\frac{bcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{1-c^4x^4}}{c^2x^2+1}+b\log\left(c^2x^3+x\right)-b\log\left(c^2x^2+cx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{1-c^4x^4}+1\right)+b\sqrt{1-c^4x^4}\operatorname{csch}^{-1}(cx)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(a*Sqrt[1 - c^4*x^4] + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4])/(1
+ c^2*x^2) + b*Sqrt[1 - c^4*x^4]*ArcCsch[c*x] + b*Log[x + c^2*x^3] - b*Log[
1 + c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/(2*c^4)
```

Maple [F] time = 0.372, size = 0, normalized size = 0.

$$\int x^3\left(a+b\operatorname{arccsch}(cx)\right)\frac{1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\left(\frac{\left(c^4x^4-1\right)\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}c^4}-2\int\left(x^3\log(c)+x^3\log(x)\right)e^{\left(-\frac{1}{2}\log\left(c^2x^2+1\right)-\frac{1}{2}\log(cx+1)-\frac{1}{2}\log(-cx+1)\right)}dx-2\int\frac{}{2\left(\sqrt{c^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*((c^4*x^4 - 1)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x
 + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate((x^3*log(c) + x^3*log(x))*e^(-1/2*l
og(c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 2*integrate(1
```

```
/2*(c^2*x^3 - x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2 + sqrt
(c*x + 1)*sqrt(-c*x + 1)*c^2), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

Fricas [B] time = 2.85791, size = 571, normalized size = 4.39

$$-\frac{2\sqrt{-c^4x^4+1}bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\sqrt{-c^4x^4+1}\left(bc^2x^2+b\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(bc^2x^2+b\right)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)+\left(\cdots\right.}{4\left(c^6x^2+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*sqrt(-c^
4*x^4 + 1)*(b*c^2*x^2 + b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) - (b*c^2*x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + (b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c
^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 2*sqrt(
-c^4*x^4 + 1)*(a*c^2*x^2 + a))/(c^6*x^2 + c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

3.177 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{a+b\,\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi [A] time = 0.0906919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A] time = 0.385569, size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Maple [A] time = 0.464, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x}\frac{1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a\left(\log\left(\sqrt{-c^4x^4+1}+1\right)-\log\left(\sqrt{-c^4x^4+1}-1\right)\right)+b\int\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{\sqrt{-\left(c^2x^2+1\right)(cx+1)(cx-1)}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(
c*x - 1))*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^4x^4+1}(b\operatorname{arcsch}(cx)+a)}{c^4x^5-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{-(cx-1)(cx+1)\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

3.178 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi [A] time = 0.103848, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A] time = 3.30896, size = 0, normalized size = 0.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Maple [A] time = 0.454, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arccsch}(cx)}{x^5}\frac{1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\left(c^4\log\left(\sqrt{-c^4x^4+1}+1\right)-c^4\log\left(\sqrt{-c^4x^4+1}-1\right)+\frac{2\sqrt{-c^4x^4+1}}{x^4}\right)a+b\int\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{\sqrt{-\left(c^2x^2+1\right)(cx+1)(cx-1)}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x
))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^4x^4+1}(b\operatorname{arcsch}(cx)+a)}{c^4x^9-x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```